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BULLETIN NO.2

1921

Section No.4

Papers on Current Transformers

by

H. W. Price and C. Kent Duff

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PAPERS ON CURRENT TRANSFORMERS

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EFFECTS OF MAGNETIC LEAKAGE IN CURRENT TRANSFORMERS

H. W. Price,
Professor of Electrical Engineering
C. Kent Duff,
Research Assistant

Introduction

In power transformers and instrument voltage transformers magnetic leakage and resulting reactance are of importance because of effects on regulation and other properties. In many types of current transformer leakage reactance is vastly greater in relative importance than in any constant-voltage transformer, being comparable in magnitude with the secondary burden. It is remarkable that magnetic leakage has usually been passed over briefly when mentioned at all in articles on current transformers.

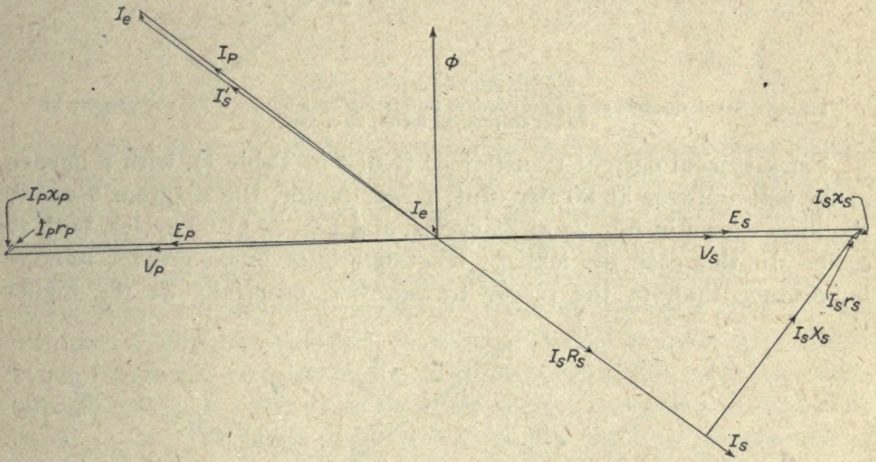

Fig. 1.—Vector diagram of an average power transformer with 1 per cent. resistance, and 5 per cent. reactance drop.

The general transformer theory is usually explained by the aid of the vector diagram shown in Fig. 1 in which;

I_s is the secondary current;

V_s is the secondary terminal voltage equal to $I_s R_s + I_s X_s j$, where R_s and X_s are the resistance and reactance respectively of the load;

E_s is the secondary induced voltage equal to $V_s + (I_s r_s + I_s x_s j)$, where r_s and x_s are the resistance and reactance respectively of the secondary winding of the transformer;

E_p is the component of primary terminal voltage consumed by the primary induced voltage which is E_s multiplied by the turns-ratio;

ϕ is the mutual flux or working flux in the core, which induces voltages in the primary and secondary windings;

I_e is the exciting current necessary to produce the flux ϕ;

I_p is the primary current, being the vector sum of I_e and I_s', the reversed secondary current;

V_p is the primary terminal voltage equal to $E_p + (I_p r_p + I_p x_p j)$.

In Fig. 1 all voltage and current vectors are drawn to scale to represent rated-load values for an average power transformer with 1 per cent. resistance and 5 per cent. reactance drop.

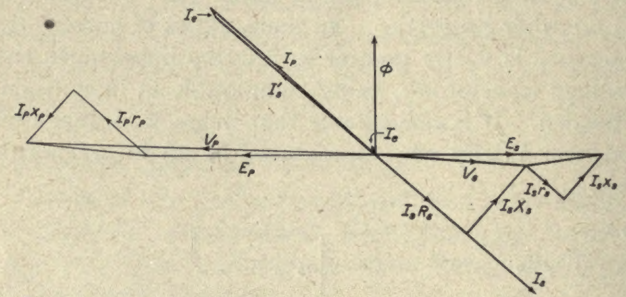

Fig. 2*a*.—Vector diagram of a typical current transformer with burden of 25 volt-amperes at 80 per cent. power factor, 25 cycles.

For a typical current transformer (No. 4 in Table I), with a burden of 25 volt-amperes at 80 per cent. power factor, the diagram assumes the form shown in Fig. 2*a* at 25 cycles, and Fig. 2*b* at 60 cycles, both to scale. In this case the leakage reactance is of much greater relative importance than in the power transformer, especially at the higher frequency. The quantities represented by light lines are seldom considered in the current transformer as they do not affect the current ratio and phase angle.

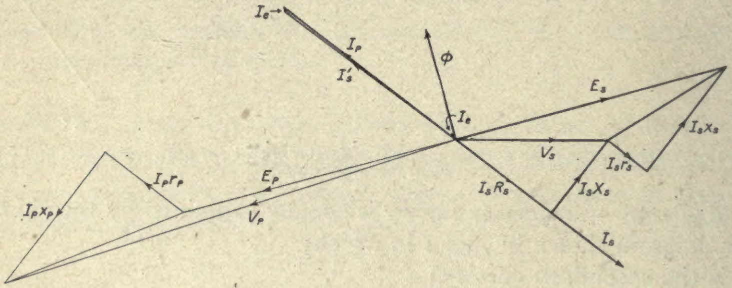

Fig. 2*b*.—Vector diagram of a typical current transformer with burden of 25 volt-amperes at 80 per cent. power factor, 60 cycles.

Further, in the power transformer, the impedance drops indicated by the triangles in Fig. 1 decrease with decreasing load, not only in absolute value but also in relative importance to the primary and secondary voltages which remain practically unchanged. In the current transformer, however, all voltages vary together, the burden being constant, and hence the leakage reactance is just as important at light loads as at full load.

The quantities in Fig. 2 which are affected by magnetic leakage are the primary leakage reactance, x_p; the secondary leakage reactance, x_s; and the core flux, ϕ. The secondary leakage reactance influences the secondary induced voltage, hence the flux, hence the exciting current, and hence the primary current. The core flux, which is assumed to be of constant value and uniform distribution in the core, is also affected by the superposition of leakage fluxes as will be explained later, and indirectly affects the exciting current and the primary current. The primary leakage reactance does not directly affect the primary or secondary currents, but the primary and secondary leakage reactances are more or less interdependent, and a change in one generally involves a change in the other.

LEAKAGE REACTANCE

In order to give a definite conception of the magnitude of leakage reactance in a number of typical current transformers Table I has been prepared. The transformers chosen are all commercial switchboard or portable types, or especially constructed transformers which are copies of commercial designs. Excepting the transformer with the ring core and uniformly distributed secondary, these figures show that in most commercial designs secondary leakage reactance is quite appreciable, being frequently comparable in magnitude with the rated secondary burden; while in a few cases the rated secondary burden is small compared to it. The latter result will evidently occur in transformers of large primary current rating, such as furnace busbar transformers, if the secondary is not distributed as on a circular core or on the four sides of a rectangular core. Other things being equal, the leakage reactance will vary as the square of the number of turns in the winding and directly as the frequency.

The primary leakage reactance is also given in Table I except in the case of through-type transformers, in which the portion of primary conductor to be considered as the primary winding is indefinite. These figures show that in a general way primary and secondary leakage reactances increase or decrease together, but it will be noticed that the total reactance is not always equally divided between the two windings as is often assumed, and in some cases the division is far from equal.

TABLE I

C.T. No.	Plate No.	Nom. Ratio	Max. Line Voltage	Type	Core	Windings	Sec. Turns	Rated Sec. Burden (Ohms)	Sec. Internal Resistance	Sec. Leakage Reactance at 60 ~	Prim. Leakage Reactance at 60 ~ in Terms of Secondary
1	1	5/5	3500	Similar to commercial switchboard types	Shell-type	Superposed link-shaped coils around middle leg of core	223	1.0	0.35	0.54	0.75
2	2	5/5	3500	"	"	Link-shaped secondary, long rectangular primary	223	1.0	0.35	1.14	5.2
3	3	75/5	15000	Switchboard	Rectangular	"	296	1.6	0.46	2	9
4	4	5/5	8000	"	"	Superposed link-shaped coils around one leg of core	222	1 to 2	0.33	0.87	1.1

TABLE I.—*Continued.*

5	5 and 6	1600/5	3500	"	"	Two link-shaped secondary coils on opposite legs of core. Single turn primary	317	1 to 2	0.32	0.86 to 1.07 †	*
6	7	1200/5		Similar to portable or switchboard types	Circular	Secondary uniformly distributed. Single-turn primary	238	1.0	0.25	Small ‡	*
7	8	1600/5	500	Portable	Rectangular	One link-shaped secondary coil on one side of core. Single-turn primary	320	0.4	0.39	3 to 7 †	*

---

*Values of primary leakage reactance in through-type transformers are not given because the portion of primary conductor to be considered as the primary winding is indefinite.

†Depending on the position of the return primary conductor.

‡The secondary leakage reactance may be positive or negative depending upon the arrangement of the primary conductor, but in all cases is quite small.

LEAKAGE FLUX

Leakage reactance is the result of leakage flux; that is, flux which fails to pass through all the turns of both windings due to the strong opposing magnetomotive forces of the two windings. Since leakage reactance is in many cases of very considerable magnitude it may be expected that leakage flux is also worthy of notice. It will be seen that if the primary and secondary windings of a current transformer can be made to produce exactly equal and opposite magnetomotive forces, there will be no mutual flux (or working flux) but leakage fluxes will be present practically the same as in normal operation. The distribution of the leakage fluxes can then be explored without interference from the working flux.

Convenient conditions for this experiment are found in a current transformer of one-to-one ratio, with equal turns in the two windings. In other cases the primary winding must be replaced by another winding of identical form and dimensions but having the same number of turns as the secondary. The new primary winding, when producing the same magnetomotive force as the original, will cause the same flux conditions in the transformer. The primary and secondary windings, when connected in series with opposite polarity, will carry the same current and will produce equal and opposite magnetomotive forces as required. Exploring coils of one or more turns may then be used in conjunction with a suitable voltage indicating instrument to determine the magnitude and phase of the flux at any point. The instrument used in the experiments here described was a separately excited suspension dynamometer with mirror, deflections being read by telescope and scale. The moving coil had a resistance of about 1,000 ohms, and the sensibility of the instrument with full field excitation was about 500 microvolts per millimetre at 1 metre distance.

The distribution of leakage flux was in this way investigated in a number of different types of current transformer under various conditions. Single-turn exploring coils were located at intervals around the core, and in some cases the core was divided up into three sections in the plane of the laminations for the placing of exploring coils. The results of these measurements are given in more detail further on, but in general it may be stated:—

(1) that both primary and secondary leakage fluxes utilize parts of the core as part of the leakage path;

(2) that the maximum primary leakage flux in the core is found at the core section which is most influenced by the primary winding, and similarly that the maximum secondary leakage flux in the core is found at the section of the core which is most influenced by the secondary winding;

(3) that the leakage flux in the core varies in magnitude at points between the two maxima, and at one point which is influenced equally by the two windings the leakage flux is zero;

(4) that there is more leakage flux in the outer laminations of the core than in the inner;

(5) that the leakage fluxes are nearly in phase with the currents in the respective windings, but that they lag by a small angle (of the order of magnitude of one or two degrees in the cases observed);

(6) that the leakage fluxes are practically proportional to the magnetomotive forces of the respective windings, or in other words, for a given current transformer the leakage fluxes vary only with the currents in the respective windings;

(7) that the leakage fluxes in their path through the core pass through the laminations not only in the normal direction along the length of the core, but also transversely, and even at right angles to the plane of the laminations.

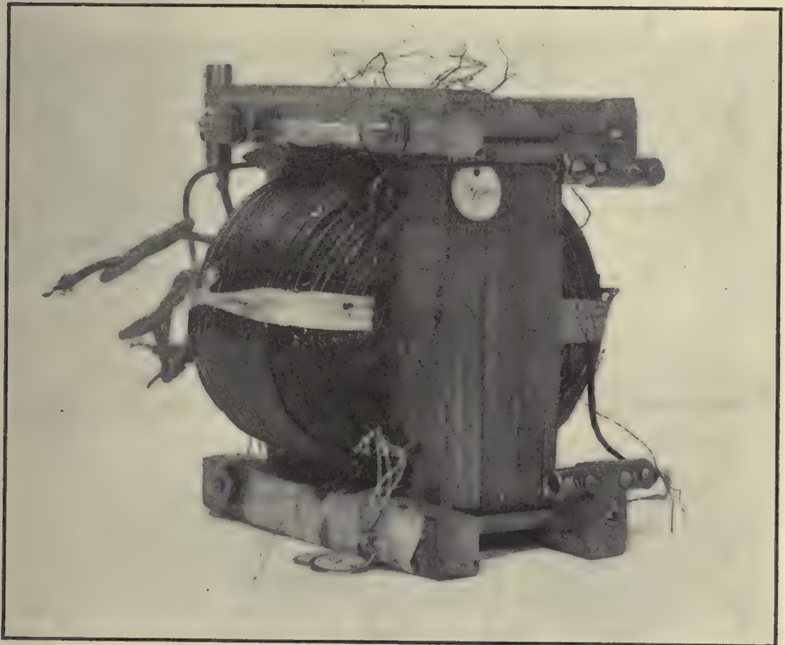

Plate No. 1.

These points may be illustrated by current transformer No. 1 of Table I, which is shown in Plate No. 1. In this case the exploring coils were located not only around the whole core but also around the individual thirds of the core, thus dividing it into three parts in the plane of the laminations as indicated in Fig. 3*a*. Fig. 3*b* shows a section through the windings and indicates the paths of a few typical lines of leakage flux. Fig. 3*c* shows vectorially the flux values observed. Vectors D to J are double the actual values which they represent because the core area at these sections is only half that at sections A to C. It will be noticed that the leakage flux in either of the outer sections of the core is approximately twice as much as that in the inner section. The maximum value of primary and secondary leakage fluxes in the core in this case was about 8,000 lines, the current being 6 amperes. That the leakage fluxes are nearly proportional to and in phase with the current in the windings is due to the fact that the reluctance of the leakage path is mostly in air, although the effect of the iron loss is noticeable in the small angle by which the current leads the leakage flux. Thus the true leakage reactance includes a small fictitious resistance due to the iron loss caused by the leakage flux.*

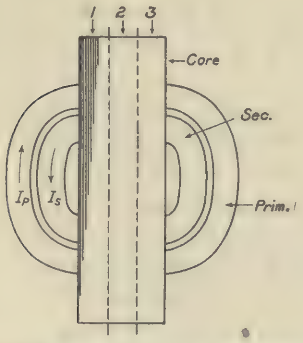

Fig. 3a.—Elevation of current transformer No. 1, showing division of core into three sections.

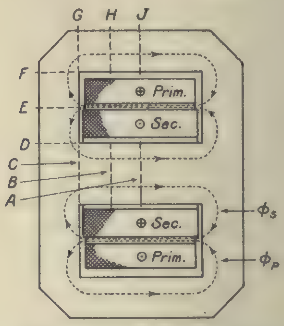

Fig. 3b.—Section of C.T. No. 1, indicating exploring coil positions and typical lines of leakage flux.

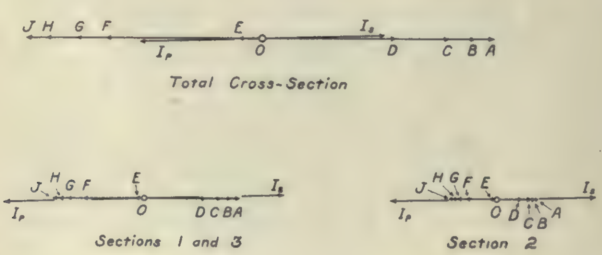

Fig. 3*c*.—Leakage flux observed in the core of C.T. No. 1, at sections shown in Fig. 3*b*.

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*Steinmetz, *Theory and Calculation of Electric Circuits*, sec. 114, page 224.

TOTAL RESULTANT FLUX

The actual flux existing in the core of many current transformers during normal operation is found to be widely different in different parts of the core. There are variations not only in magnitude and phase, but also in distribution over the core cross-section.

This is illustrated in the case of transformer No. 1 of Table I, before referred to, by the vector diagrams in Fig. 4*a* which represent the flux in the core with the primary carrying 6 amperes at 60 cycles with a burden of 0.99 ohms non-inductive resistance. Comparing these diagrams with those of Fig. 3*c*, it will be noticed that the total flux at each point is the vector sum of the leakage flux originally observed at that point plus a constant component ϕ_m. The latter is obviously the working flux or mutual flux which is said to cause in both primary and secondary windings a "nominal induced voltage" proportional to the turns in each.

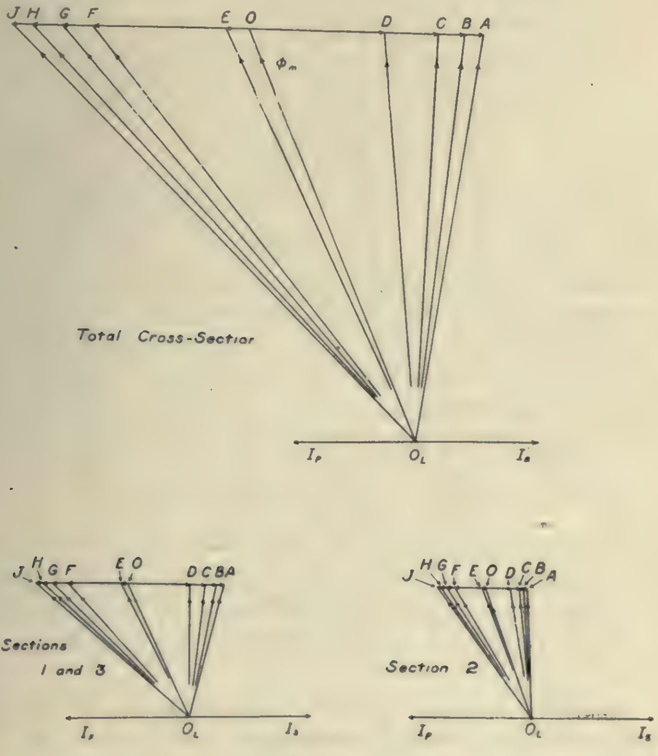

Fig. 4*a*.—Flux vector diagrams of C.T. No. 1, with 0.99 ohm, non-inductive burden at 60 cycles.

The secondary induced voltage, E_s, Fig. 4*b*, lags 90 degrees with regard to the working flux, and forces the secondary current through the total impedance of the secondary circuit. The component of E_s in phase with I_s overcomes the resistance of the circuit, while the quadrature component provides for the reactance drop. These two components, E_r and E_x, may be considered as due to ϕ_r and ϕ_x respectively, the two components of the working flux with regard to I'_s. Thus ϕ_r is proportional to the total resistance in the secondary circuit and ϕ_x to the total reactance.

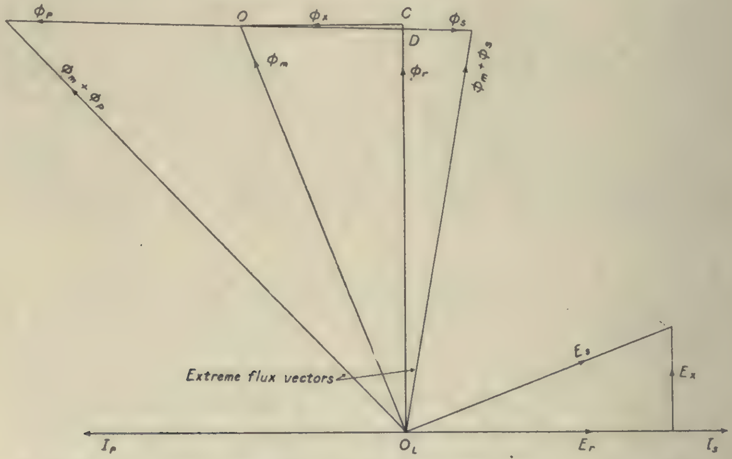

Fig. 4b.—Transformer diagram of C.T. No. 1, showing components of ϕ and Es.

In this particular case the resistance of the burden was 0.99 ohm and of the secondary winding 0.35 ohm, and their sum, 1.34 ohms, is proportional to $O_L D$ in Fig. 4b, while DC represents the small fictitious resistance due to the iron loss caused by the leakage flux. OC represents the effective leakage reactance since no other reactance existed in the secondary circuit. The secondary leakage reactance is therefore equal to $1.34 \times \frac{OC}{DO_L}$ which is found to be 0.54 ohm in this case.

Thus the leakage reactance is determined as a by-product of an investigation of flux distribution. It is unnecessary to know the calibration constant of the dynamometer if the resistance of the secondary winding and the resistance and reactance of the secondary burden are obtainable.

It is interesting to notice how the flux diagram of Fig. 4b will be influenced by changing various conditions. Increase in the resistance of the burden moves the point O_L vertically downward. Increase in the reactance of the burden moves the points O_L and C horizontally to the right. Increase in current with constant secondary burden* enlarges

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*Constant burden meaning in this case constant resistance and *inductance*.

the whole diagram in proportion. Increase in frequency with constant burden* and current moves the point O_L vertically upward.

As already stated, the leakage flux is greater in the outer laminations of the core, and in transformer No. 1 was found to be twice as great in either of the two outside sections as in the inner section. The working flux is more uniformly distributed over the cross-section, but is nevertheless noticeably greater in the outer laminations. This is probably due to the fact that the leakage flux density in the outer laminations is greater, thus increasing the permeability of this part of the core, it being kept in mind that the density was below the point of maximum permeability of the iron, as is usual in well-designed current transformers.

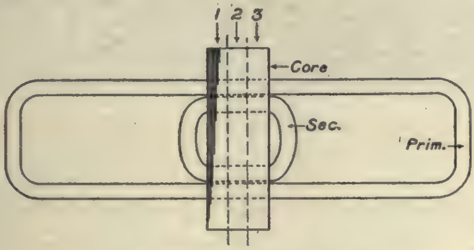

Fig. 5*a*.—Sketch of C.T. No. 2.

The flux distribution in other transformers of Table I will be given with briefer comment.

Fig. 5*a* is a sketch of transformer No. 2, which is shown in Plate No. 2. Fig. 5*b* represents the flux in different parts of the core. Transformer No. 2 is No. 1 with the previous primary coil replaced by another of long rectangular form. Plate No. 3 of transformer No. 3 shows a commercial transformer with this style of primary winding. This increases both primary and secondary leakage flux and leakage reactance, and further changes the ratio of secondary to primary leakage reactance from the proportions 1:1.39 to 2.1:9.6. Also the ratio of the leakage flux in the outer sections of the core to that in the inner section

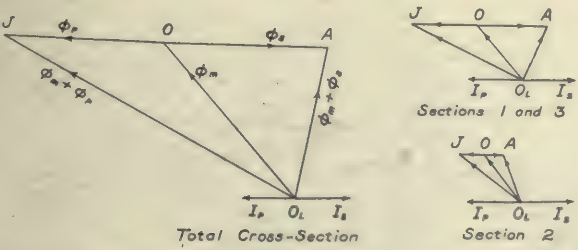

Fig. 5*b*.—Flux vector diagrams of C.T. No. 2 with 0.99 ohm non-inductive burden at 60 cycles.

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* Constant burden meaning in this case constant resistance and *inductance*.

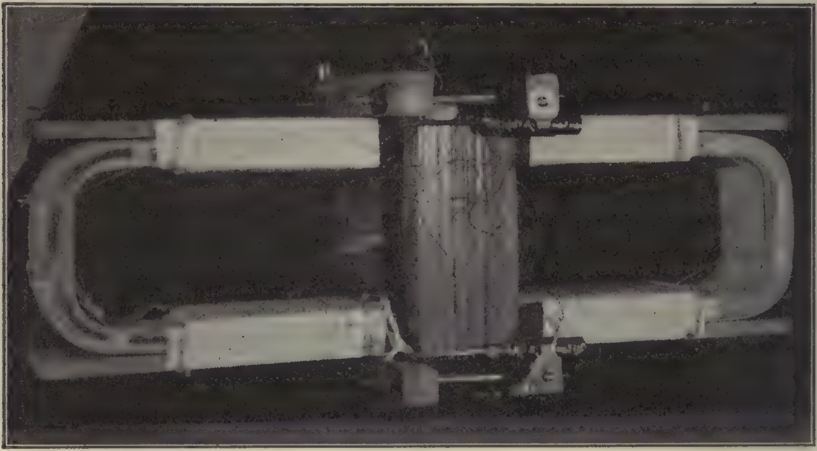

Plate No. 2.

becomes approximately three to one instead of two to one as in transformer No. 1. Conditions for transformer No. 2 were:—

I_p = primary current = 6 amperes.

F = frequency = 60 cycles.

r_s = secondary internal resistance = 0.35 ohm.

R_s = secondary external resistance = 0.99 ohm.

x_s = secondary internal reactance = 1.14 ohm.

X_s = secondary external reactance = 0.

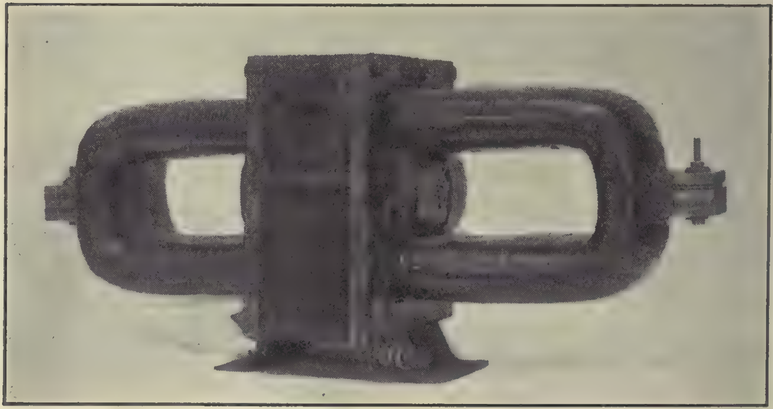

Plate No. 3.

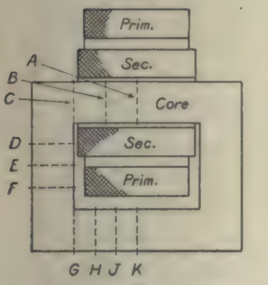

Fig. 6a.—Section of C.T. No. 4, showing exploring coil positions.

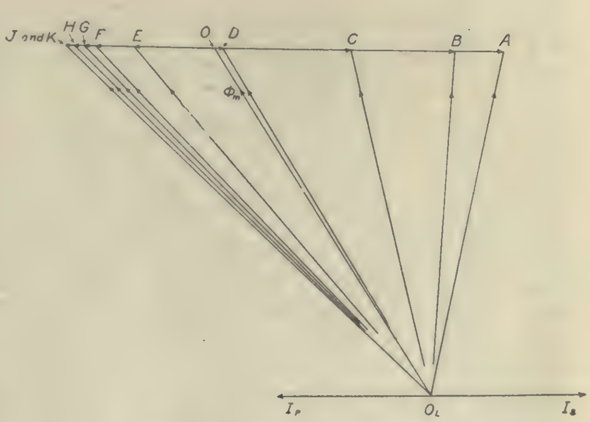

Fig. 6b.—Flux vector diagram of C.T. No. 4.

Fig. 6*a* shows the arrangement of the windings and the exploring coil positions on transformer No. 4, Plate No. 4, with the flux distribution as shown in Fig. 6*b* for the following conditions:—

$I_p=6$ $r_s=0.33$ $x_s=0.87$
$F=60$ $R_s=0.99$ $X_s=0.$

Transformer No. 5 is shown in Plate No. 5 with a distributed primary winding which will give an approximation to the effect of a straight through conductor. On the right of the picture is seen the metal case and insulating tube with which the transformer is provided

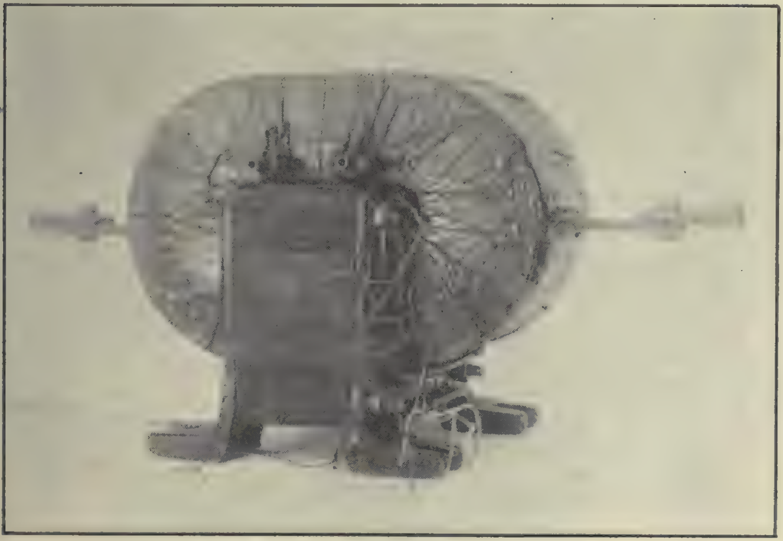

Plate No. 4.

for installation on a busbar. The exploring coil positions are indicated in Fig. 7*a* and the flux distribution in Fig. 7*b*. Conditions are as follows:

$I_p = 6$ $r_s = 0.32$ $x_s = 0.96$.
$F = 60$ $R_s = 0.99$ $X_s = 0$.

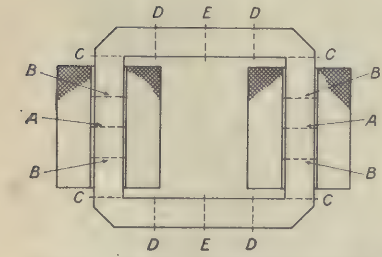

Fig. 7*a*.—Section of C.T. No. 5, showing exploring coil positions.

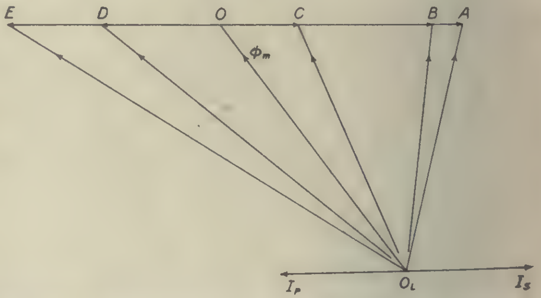

Fig. 7*b*.—Flux distribution in core of C.T. No. 5, with 0.99 ohm non-inductive burden, 60 cycles.

This transformer, with other positions of the primary, has secondary leakage reactances at 60 cycles varying from 0.86 to 1.07 ohm, the former value being obtained with half of the primary passing around each secondary coil as in Plate No. 6 and the latter with half of the primary passing around each end of the core. These two conditions would correspond to the use of two or more turns of cable in a portable through-type transformer of this construction. With a metal case the

Plate No. 5.

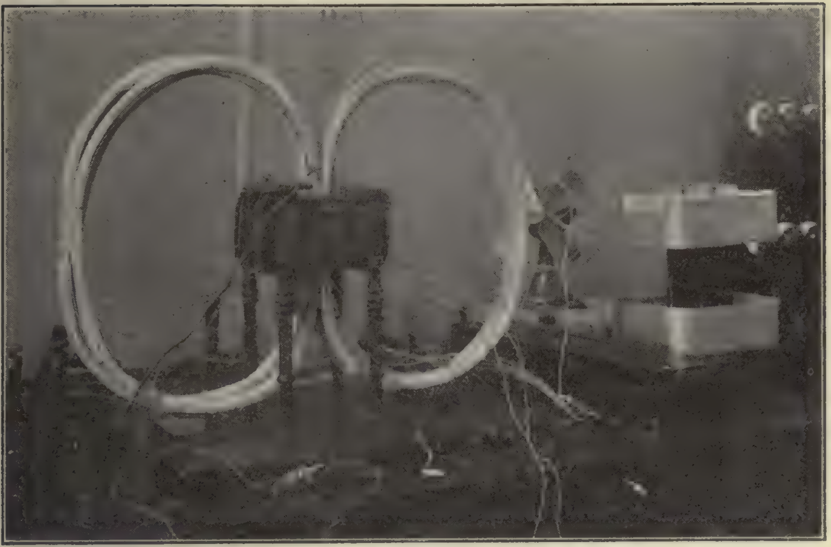

Plate No. 6.

flux distribution and leakage reactances in this transformer would probably be appreciably different.

Transformer No. 6 is a style sometimes used in through-type switchboard and portable current transformers. With a symmetrical well-distributed primary, or its equivalent a straight through conductor with return conductor at a considerable distance, no leakage flux enters the

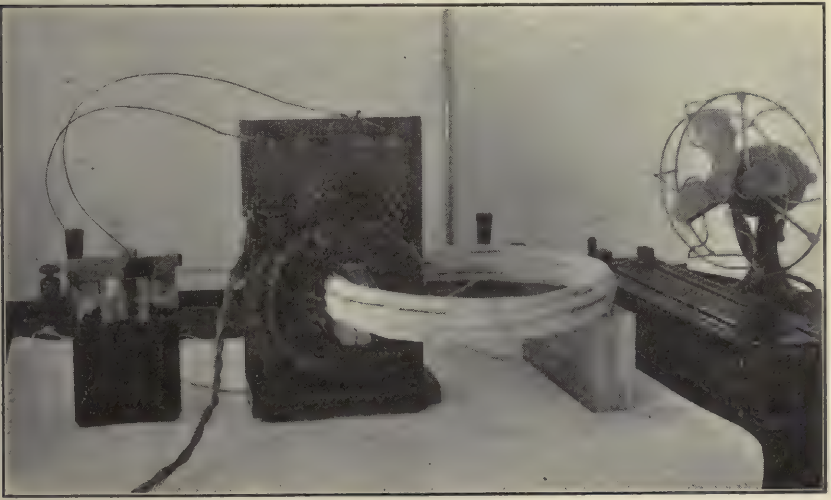

Plate No. 7.

core, and the core flux is the same at all cross-sections. In actual switchboard installations, and in use as a portable transformer, the primary is seldom completely distributed; and in such cases the core flux is not uniform. In Plate No. 7 of this transformer the primary winding is shown as a circular loop about a foot in diameter passing around one side of the core. With this primary and normal operating conditions the leakage flux in the core and the total flux were found to be as ahown in Fig. 8*b*. The working flux and leakage fluxes must, therefore, travel somewhat as suggested in Fig. 8*a*. It will be seen that the working flux, when resolved into its components with regard to the secondary current, appears to indicate a *negative reactance* in the secondary circuit. The resistance of the secondary was 0.25 ohm, and the burden in this case was 0.99 non-inductive resistance. Since the portion of the secondary circuit external to the transformer is non-inductive the reactance shown by Fig. 8*b* must be that of the secondary winding, and further must be negative.

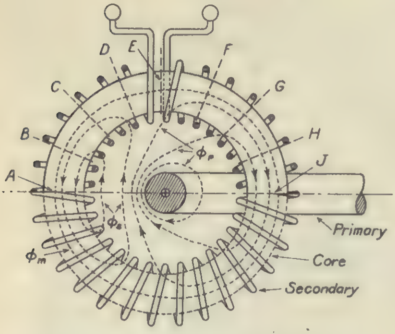

Fig. 8*a*.—Section of C.T. No. 6, showing. exploring coil positions and leakage paths.

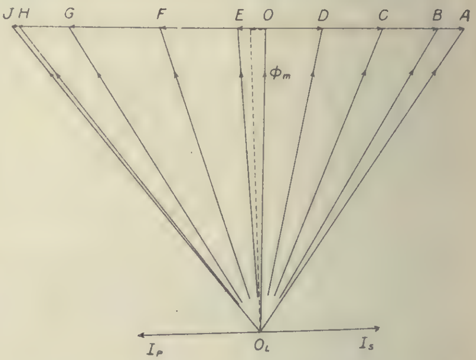

Fig. 8*b*.—Flux distribution in core of C.T. No. 6, with 0.99 ohm non-inductive burden, 60 cycles.

An explanation of this negative leakage reactance is to be found in the fact that the primary leakage flux in passing through the core also passes through some turns of the secondary winding which is uniformly distributed on the core. These primary leakage lines linking with secondary turns, tend to neutralize the reactive effect of the secondary leakage lines which are opposite in sense; and since a greater part of the core carries primary leakage flux, and also because the primary leakage flux density is greater the integrated linkages of primary leakage lines with secondary turns is greater than that of secondary leakage lines with secondary turns, with the result that the secondary reactance appears to be negative.

The question may be asked whether flux which passes through part of both primary and secondary turns is not really working flux or mutual flux. It is true that such flux is strictly mutual flux with regard to the parts of each winding to which it is common, but the leakage fluxes mentioned above differ in several important respects from the ordinary working flux which is common to all turns of both windings. The following reasons are given for considering flux which is common to the primary and only part of the secondary as leakage flux and not as working flux:

(1) Unlike working flux, it is present when equal and opposite magnetomotive forces are produced by the primary and secondary windings.

(2) Unlike working flux, its path is not completely within the core and it does not completely interlink both windings.

(3) Unlike working flux, its magnitude and phase are practically independent of the secondary burden.

(4) Like leakage flux, its magnitude and phase are determined by the magnitude and phase of the current in the windings.

This apparently negative secondary leakage reactance was confirmed by a ratio and phase angle test at 60 cycles with secondary short-circuited. It was found that even with small values of secondary current such as 0.75 and 1.0 ampere, and with equal turns in primary and secondary the observed ratio was 1.000; which shows that the exciting current was in quadrature with the secondary current. This could not have been the case unless the core losses were zero or the secondary current was leading the secondary induced voltage. Although the core loss was small at such a low density, it was not zero; which leads to the conclusion that the effective secondary leakage reactance was negative for that arrangement of primary and secondary coils.

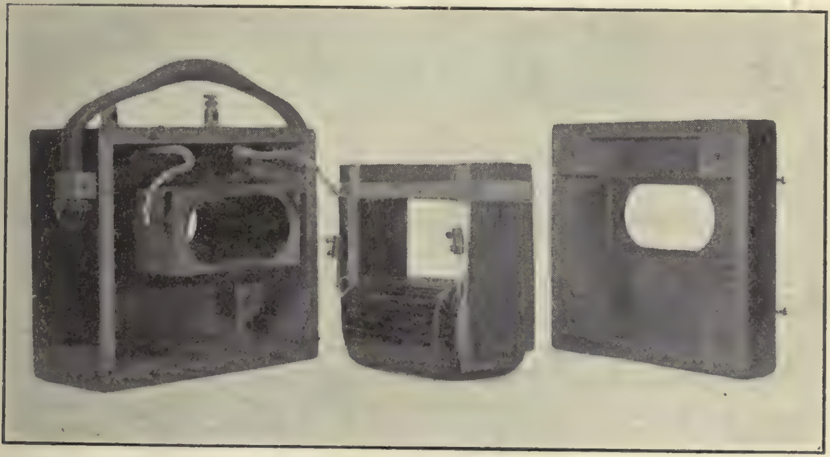

Plate No. 8.

The construction of transformer No. 7 of Table I is clearly shown in Plate No. 8. This transformer possesses the greatest leakage reactance of any thus far considered. The reactance depends very much upon the arrangement of the primary. If the primary is wound around the secondary coil in a circular loop about a foot in diameter the secondary leakage reactance at 60 cycles is about 3 ohms, while if the primary is looped around the opposite side of the core as in Fig. 9*a*, the value is 7 ohms. Fig. 9*c* indicates the flux distribution and the appearance of the transformer diagram for the latter case with the following conditions:—

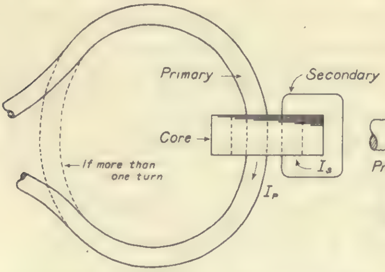

Fig. 9*a*.—Sketch of C.T. No. 7, showing position of primary.

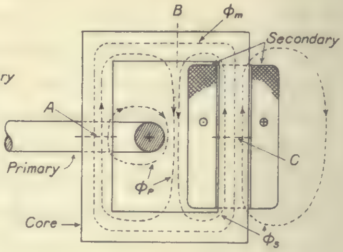

Fig. 9*b*.—Section of C.T. No. 7.

$I_p = 2.6$ $r_s = 0.4$ $x_s = 7$
$F = 60$ $R_s = 0.84$ $X_s = 0.$

Notice that the actual flux $\phi_m + \phi_p$ present at section A is approximately eight times that at section C, and about 75 degrees out of phase with it. In this case, due to the low power factor of the secondary circuit, the secondary current lags with regard to the primary current, and there is a corresponding phase difference in the leakage fluxes ϕ_p and ϕ_s.

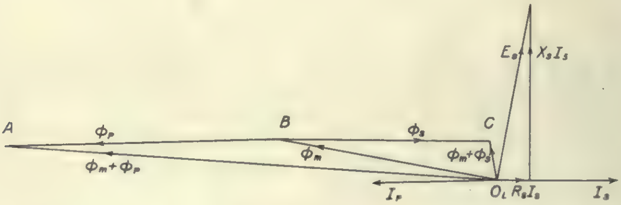

Fig. 9*c*.—Transformer diagram and flux distribution of C.T. No. 7.

EFFECT OF LEAKAGE FLUXES ON THE EXCITING CURRENT

Although the primary leakage flux in the core is usually of the same order of magnitude as the secondary leakage flux, and the mutual flux is, roughly speaking, midway between the two extremes of flux observed in the core, still the exciting current required by a core carrying mutual flux with superposed leakage fluxes is in general not the same as that which would be required if the same mutual flux existed without the leakage fluxes. This is shown in a very marked manner in the case of transformer No. 7 by comparing observed ratio and phase angle curves with those calculated in the usual way on the assumption of uniform flux in the core. See Fig. 10. The calculated curves were based on exciting current measurements made on the actual core of transformer No. 7, using the secondary winding as an exciting winding, the primary circuit being open. Under these conditions the flux will be practically uniform throughout the core, and not variable from point to point as observed under actual operating conditions.

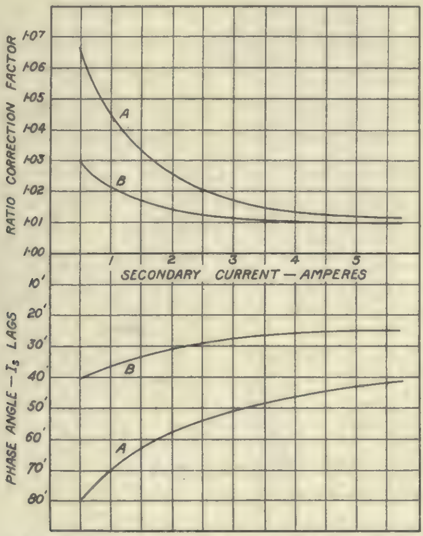

Fig. 10.—Ratio and phase-angle curves of C.T. No. 7, with primary distributed and secondary short-circuited. *A*, calculated; *B*, observed.

For the purpose of analysis the actual exciting current of a current transformer may be considered as the vector sum of the exciting currents required by a large number of small sections into which the core may be divided along its length. Any expectation of agreement between the observed ratio and phase angle curves and those calculated as outlined above is based on the general assumption that the exciting current under the flux conditions existing in actual operation will be the same as that which would be required if the same mutual flux existed without the leakage fluxes. This general assumption involves the following particular assumptions, many of which are shown to be untrue.

(1) It is assumed that the mutual flux is the vector average of the maximum and minimum fluxes observed in the core. In no observed

case has this been found exactly true although the mutual flux is always somewhere between the maximum and minimum fluxes.

(2) It is assumed that half of the core is affected by primary leakage flux, and the other half by secondary leakage flux, and to an equal extent. This assumption has also been found to be inaccurate in all observed cases, although it holds in a very rough manner.

(3) It is assumed that the flux in the core is uniformly distributed over the core cross-section, and travels in a path parallel to the core axis. As has been stated and shown in the diagrams, the flux is not uniformly distributed over the cross-section, nor does it travel in the normal direction, *i.e.*, parallel to the core axis.

(4) It must be assumed that the core reluctance is constant in order that the lower magnetizing current required by the sections of the core in which the flux is less than the average may be exactly counterbalanced by the higher magnetizing current required by the other parts of the core. Actually the core reluctance of the iron varies considerably with the flux density, well designed current transformers operating normally at maximum crest densities below the point of maximum permeability of the iron and below the straight part of the saturation curve.

(5) It is assumed also that the core-loss current varies directly with the density in all parts of the core. Otherwise the total exciting current for all parts of the core could not be that corresponding to an assumed uniform average flux density. If the core-loss current is to vary directly with the flux density, then the core losses must vary directly as the square of the density. The eddy loss does vary as the square of the density because eddy currents vary as eddy voltages, which in turn vary as the density at given frequency. Usually, however, 50 to 80 per cent. of the core-loss is due to hysteresis. This part of the loss usually does not vary as the square of the density and since it is the major part of the core-loss the assumption under discussion can be justified only as an approximate one.

With these facts in view it is not surprising that the calculated and observed characteristics do not agree. The complexity of the problem renders it exceedingly difficult to formulate a correction factor which will make the calculated results agree with the observations.

The discrepancies are not usually so great as in transformer No. 7, which has a very large leakage reactance. Fig. 11 gives a comparison of the observed and calculated curves of transformer No. 5 with distributed primary. Fig. 12 shows the results on transformer No. 4. The discrepancies in these two cases might be considered negligible for many purposes. A greater discrepancy might easily be caused by a less accurate knowledge of the value of the secondary leakage reactance than was obtained in these experiments. Ordinarily the secondary leakage reactance would be computed from the physical constants of the windings by the aid of some empirical formula, or estimated by making an arbitrary division of the total reactance of the transformer into primary and secondary reactances, by which means only an approximate idea of the secondary reactance can be obtained. However, it seems improbable that the discrepancies observed in these experiments are due to errors in the determination of the secondary leakage reactance. They must, therefore, be due to the effects of leakage flux in the core as before explained.

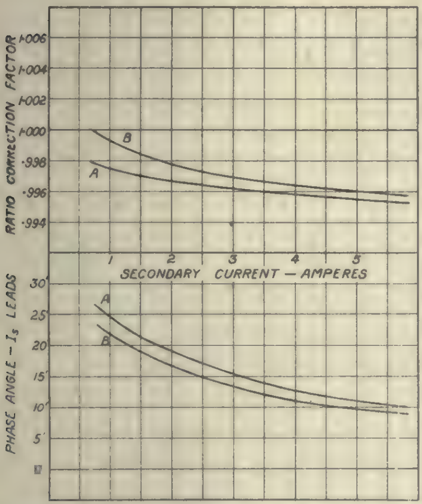

Fig. 11.—Ratio and phase-angle curves of C.T. No. 5, with distributed primary and 0.99 ohm non-inductive burden, 60 cycles. *A*, calculated; *B*, observed.

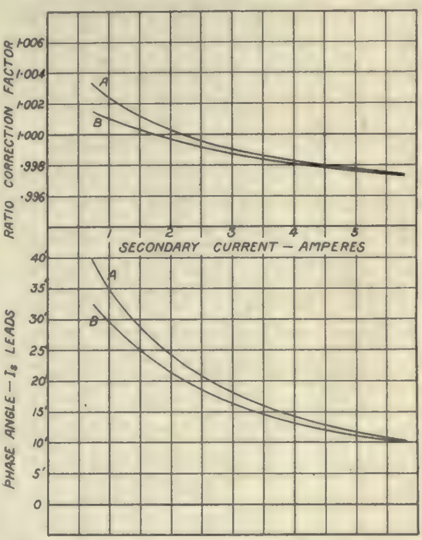

Fig. 12.—Ratio and phase-angle curves of C.T. No. 4, with 0.99 ohm non-inductive burden, 60 cycles. *A*, calculated; *B*, observed.

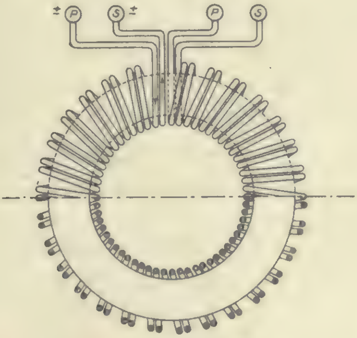

Fig. 13.—C.T. No. 6, with uniformly distributed primary winding.

This statement is supported, though not absolutely proved, by an experiment made on transformer No. 6. This transformer has two identical core windings of 119 turns each, both uniformly distributed around the core, and arranged as shown in Fig. 13 so that each turn of one winding has by its side a turn of the other winding. One of these windings may be used as the primary and the other as the secondary of a one-to-one current transformer. This construction was chosen to make the magnetic leakage small, and to prevent as far as possible those leakage lines which did exist from entering the iron core. From the arrangement of the windings, all leakage flux must pass through the narrow space occupied by the insulation between turns. The leakage reactance is very small, about 0.002 ohm per winding at 60 cycles, and no perceptible leakage flux enters the core as would be expected from the construction. The observed and calculated ratio and phase angle curves of this transformer are practically coincident, the maximum discrepancy between individual points being 1/20 of one per cent. in ratio and 2 minutes in phase angle from one-fifth to full load current.

In order to determine in more detail the effects of leakage flux on the exciting current the components of the latter were obtained by inverse calculation from observed ratio and phase angle curves of various transformers. These results were compared with the exciting current measured with uniform flux as before described. From a large number of calculations of this kind, on transformers of different types, few conclusions can be drawn to which there are no exceptions. However, the following statements may be made:—

(1) In a current transformer in which no leakage flux enters the core the ratio and phase angle may be accurately calculated in the usual way, *i.e.*, from measurements of core-loss and magnetizing currents made with uniform flux distribution, a knowledge of the turn-ratio, and the constants of the secondary circuit. The accuracy of the calculated results depends only on the accuracy of the data from which they are computed.

(2) In a current transformer in which leakage flux exists in the core, superposed on the working flux, the ratio and phase angle can not be calculated with consistent accuracy by the usual method referred to above.

(3) The magnitude of the discrepancy increases with the magnitude of the leakage flux in the core, or in other words with the leakage reactance, other things being equal.

(4) The calculated ratio is usually too high and the calculated angle of lead of I_s with regard to I_p is usually greater than the actual.

(5) The actual core-loss current may be higher or lower than core-loss current measured with uniform flux.

(6) The actual magnetizing current is lower than that measured with uniform flux over a considerable range of densities below the density corresponding to maximum permeability of the iron.

This last mentioned point appears to be the explanation of the bulk of the discrepancy. The magnetizing component is usually larger than the core-loss component of the exciting current, and seems to be more affected by leakage fluxes. The reluctance of the iron is reduced by the addition of leakage flux to the mutual flux, since the densities are usually lower than that at which the permeability of the core iron is maximum.

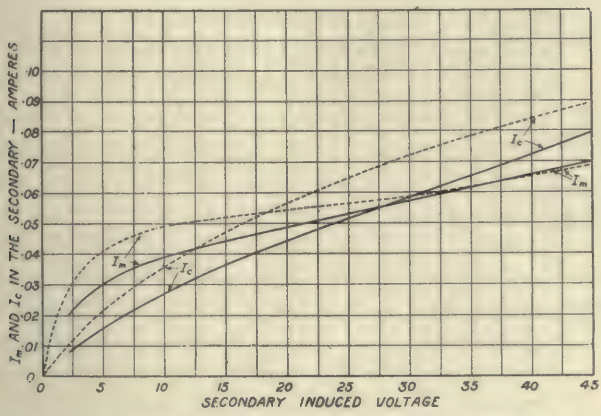

Fig. 14.—Magnetizing and core loss components of exciting current of C.T. No. 7, with primary as in Fig. 9*a*. *I*m, magnetizing component; *I*c core loss component. *Dotted lines*, by direct measurement with uniform flux in core; *full lines*, actual values when leakage fluxes are present.

As a sample of the discrepancies which are observed, Fig. 14 shows the magnetizing and core-loss components of the exciting current of transformer No. 7. The broken lines are plotted from the results of direct measurement of the exciting current with uniform flux in the core while the full lines are obtained by inverse calculation from the observed ratio and phase angle curves of Fig. 10. The discrepancies in this transformer are rather extreme since the leakage reactance is so large.

EFFECT OF LEAKAGE REACTANCE ON RATIO AND PHASE ANGLE

Although leakage reactance, accompanied by leakage fluxes in the core makes calculations of ratio and phase angle unreliable if high accuracy is desired, it does not follow that the effects on the actual ratio and phase angle curves obtained are always undesirable. These curves are affected in different ways by leakage reactance and will be considered separately.

Increase of leakage reactance in a current transformer, other conditions being unchanged, will reduce the angle of lead of I_s with regard to I_p. This is because the power factor of the total secondary circuit is decreased, causing the secondary current to lag more with regard to the secondary voltage. In most cases the power factor of the total secondary circuit will be higher than that of the core, so that increase of leakage reactance brings the secondary current more into phase with the exciting current, thus reducing the phase angle between primary and secondary currents.

Increase of leakage reactance raises the ratio, and also usually increases the slope of the ratio curve. There are two reasons for this. One is that increase of leakage reactance increases the total impedance of the secondary circuit, and an increase of the exciting current is necessary, which raises the ratio. The power factor of the secondary circuit is at the same time reduced, which causes the ratio to be more dependent on the magnetizing component of the exciting current than on the core-loss component. Since the magnetizing current is usually larger than the core-loss current, and since it is not generally so near to a linear function of the density as the core loss current at the densities used in well designed current transformers the ratio is raised due to this cause also, and the slope of the ratio curve is increased. These effects are somewhat undesirable but may be partially corrected by turns compensation.

In consideration of the reduction of the phase angle errors these effects on the ratio may be considered permissible, but if phase angle errors are considered unimportant for the purpose for which the current transformer is to be used, leakage reactance should be reduced to a minimum in order to obtain minimum ratio errors.

We are indebted to the Hydro Electric Power Commission of Ontario for the loan of certain apparatus and for other assistance.

METHOD OF MEASURING RATIO AND PHASE ANGLE

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Professor of Electrical Engineering
C. Kent Duff
Research Assistant

The methods of measuring the ratio and phase angle of current transformers in the experimental work connected with the research described in the preceding paper fall under the general classification of comparison methods.

In some cases the current transformer to be investigated is compared with a standard transformer of the same ratio by the deflection method described by F. B. Silsbee.* The primaries of the two transformers are connected in series and the secondaries are both connected to the current coil of a Weston single-phase low-power-factor wattmeter in such manner that the coil carries the difference between the two secondary currents. This difference is a vector difference. By exciting the voltage circuit of the wattmeter from two different phases, two readings are obtained which give the components of the difference-current. The ratio and phase angle of the transformer in question, with regard to the standard transformer, may then be calculated; and by correcting for the known errors of the standard transformer, the ratio and phase angle of the other are obtained.

In most of the transformers dealt with in this research the ratio was one-to-one, or was made so by using the required number of primary turns. This was necessary in order to determine the leakage flux distribution and the leakage reactance under different conditions, and incidentally it made possible a simplification of the comparison method for measuring ratio and phase angle. The secondary current may be compared directly with the primary current which may be thought of as the secondary current of a standard transformer with no errors. Corrections for the errors of a separate standard transformer are thereby avoided.†

The connections used are shown in Fig. 1. The vector difference between primary and secondary currents is i, which flows through the current coil of a wattmeter. The voltage coil of the wattmeter is supplied first from V_P and then from V_Q, two readings being obtained from which the components of i in phase and in quadrature with the primary current are calculated. V_P is in phase with the primary current since it is the voltage across the primary lamp bank which is non-inductive. V_Q is adjusted in quadrature with the primary current by rotating the phase shifter until the wattmeter in the main line reads zero.

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*Electrical World, 70, p. 667, Oct., 1917.

†Testing of Instrument Transformers, III, H. M. Crothers, *Electrical World*, 75, No. 6, Feb., 1920.

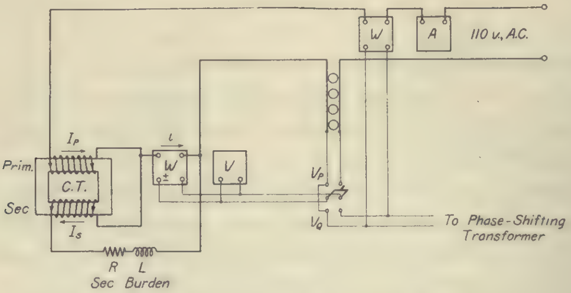


Fig. 1.—Connections for measurement of ratio and phase angle of current transformer.

The vector diagram, Fig. 2, indicates the various phase relations. From the two wattmeter readings are derived i_p and i_Q. The phase angle θ is $\sin^{-1}\frac{i_Q}{I_s}$. Unless an ammeter is included in the secondary circuit, I_s will be unknown. However, if the ratio error is small, the phase angle is approximately $\sin^{-1}\frac{i_Q}{I_p}$, or more correctly $\sin^{-1}\frac{i_Q}{I_p-i_p}$. The ratio is $\frac{I_p}{I_s}=\frac{I_p}{\frac{I_p-i_p}{\cos\theta}}=\frac{I_p\cos\theta}{I_p-i_p}$. If the phase angle is small, $\cos\theta$ is very nearly one, and the formula becomes $\frac{I_p}{I_s}=\frac{I_p}{I_p-i_p}$.

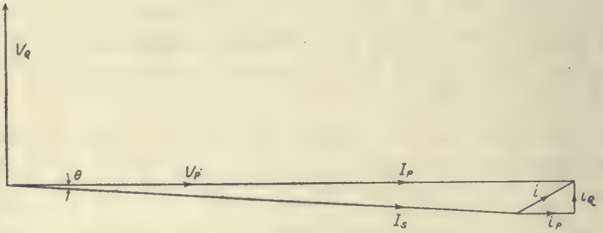


Fig. 2.—Vector diagram of currents in ratio and phase-angle measurement.

The wattmeter used for measuring the difference between primary and secondary currents was a Weston, Model 310, single-phase, low-power-factor wattmeter, 5 or 10 amperes, 75 or 150 volts, 75, 150 or 300 watts, with 150 line scale. The 5-ampere current connection was used and the 75 volt pressure circuit, although the voltages impressed were from 100 to 125 volts in order to increase the deflection. With 100 volts on the 75 volt circuit, one scale division represents 0.005 ampere in phase with the voltage, which means 0.1 per cent. in ratio or a phase angle of 3.3 minutes at full load current of 5 amperes. Readings were estimated to tenths of one division.

In the use of this wattmeter certain precautions must be observed in order to avoid errors. Although shielded, the instrument should be placed at a considerable distance from sources of stray fields such as current transformers, the phase-shifting transformer, and conductors carrying heavy currents. The over excitation of the pressure coil makes this particularly necessary. Also, connections should be so made that the pressure terminal marked ± is at the potential of the current circuit in order to avoid deflections due to electrostatic effects within the meter. This may be done by seeing that the ± pressure terminal is connected through switch V_{p} to the side of the lamp bank next the wattmeter. A connection across the double-throw switch, as shown in Fig. 1, is necessary for the same purpose in regard to V_{Q}. If the wattmeter reads negative the current leads should be reversed at the wattmeter terminals.

Silsbee points out the possibility of errors due to voltages induced in the current coil from the flux of the moving coil and suggests setting the moving coil in the position of zero mutual inductance. This was not done in the present instance because it was found that in curves which cross the zero line there is no appreciable break in continuity of the curve in spite of the fact that the current connections are reversed. This is possibly because the voltage induced in the current coil would be in quadrature with the current in the moving coil.

The impedance of the current coil of the wattmeter should be as low as possible in order to avoid any appreciable voltage drop between the points to which it is connected. Such impedance tends to make the secondary current more nearly equal to the primary current both in magnitude and phase, and therefore changes the ratio and phase angle of the transformer. In other words it changes the voltage which must be produced by the secondary, and therefore has the effect of a change in the secondary burden. The disturbing effect of the impedance of the wattmeter current coil in these measurements may be calculated from the following formula. The effective *increase* in the secondary burden due to the effect of the wattmeter stated as a complex quantity is:—

$$-\frac{i}{I_s}\left\{R_{\mathrm{w}}\sin\,(\delta+\beta)+X_{\mathrm{w}}\cos\,(\delta+\beta)\right\}+j\frac{i}{I_s}\left\{R_{\mathrm{w}}\cos\,(\delta+\beta)-X_{\mathrm{w}}\sin\,(\delta+\beta)\right\}$$

where I_s is the secondary current;

i is the vector difference between primary and secondary currents, $I_p - I_s$;

R_w is the resistance of the wattmeter current coil and leads;

X_w is the reactance of the wattmeter current coil and leads;

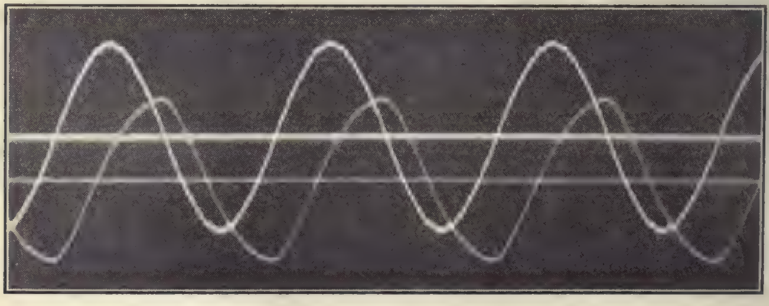
δ is the angle of lead of i with regard to ϕ, the working flux;

β is the angle of lead of E_s with regard to I_s.

If the above value is small with regard to the impedance of the secondary circuit, the effect of the wattmeter on the ratio and phase angle will be correspondingly small. In the wattmeter used, the resistance of the 5 ampere current circuit was 0.13 ohm and its inductance 0.00021 henry, and the value of the impedance calculated from the above expression was rarely more than 0.5 per cent. of the impedance of the secondary burden. By way of practical confirmation the effect of increasing the resistance of the wattmeter circuit was observed. The wattmeter reading showed no perceptible change when the impedance in the wattmeter circuit was doubled for readings up to ten scale divisions.

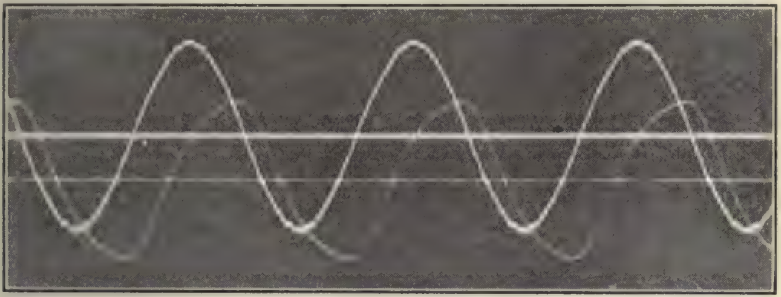
It is assumed throughout this method that all quantities are sine waves, but that is not strictly true. The primary ammeter will read the mean effective value of the primary current regardless of its wave form, but the indications of the wattmeter carrying the difference-current will depend upon the wave forms of both the current and the voltage. This difference-current in the case of an uncompensated current transformer is the exciting current of the transformer, which may therefore be expected to contain harmonics.

In this research a 60-cycle alternator was used which gave a fair approximation to a sine wave, a small tooth ripple being the only noticeable harmonic. With this alternator as supply a series of oscillograms was taken of the exciting current of a sample current transformer at various densities. The core length was 15.7 inches, net cross-section 1.94 square inches, weight 8.56 pounds, being in the form of a ring without joints. In Osc. Nos. 11 to 14 the upper wave represents the

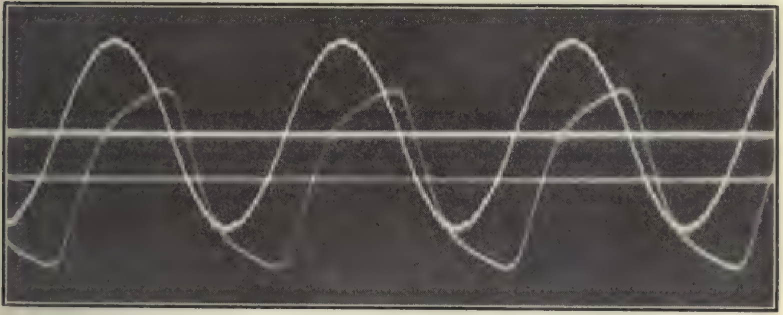


Oscillogram No. 11.

voltage applied to the exciting winding and the lower wave the exciting current, connections being as shown in Fig. 3. Notice that at the low densities the exciting current is very little distorted while with increasing density the effects of hysteresis and saturation introduce harmonics. The greatest crest density in well designed current transformers under normal operating conditions does not usually exceed 25,000 lines per square inch.



Oscillogram No. 12.



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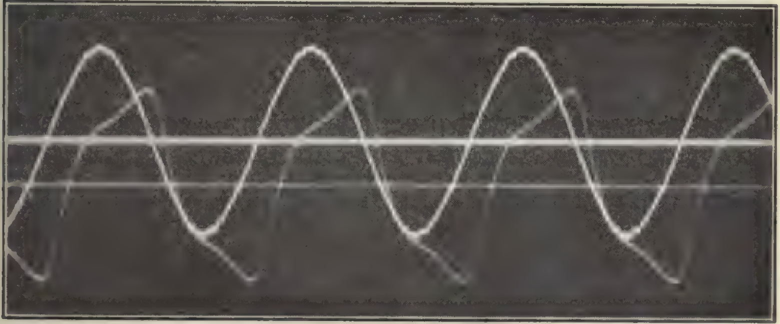


Oscillogram No. 14.

Oscillograms were also taken with connections as in Fig. 1, the elements being connected to show the wave forms of I_p, i and I_s respectively. In Osc. Nos. 17 to 20 the impedance of the secondary was $R=0.54$, $X=0.94$, which produces a power factor in the secondary circuit equal to that of the core and causes the primary and secondary currents to be in phase with the exciting current. The density is 8800 lines per square inch, which is somewhat greater than that in Osc. No. 12. In Osc. No. 17 there is no compensation, hence the difference-current is the exciting

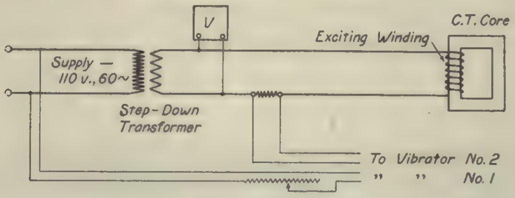
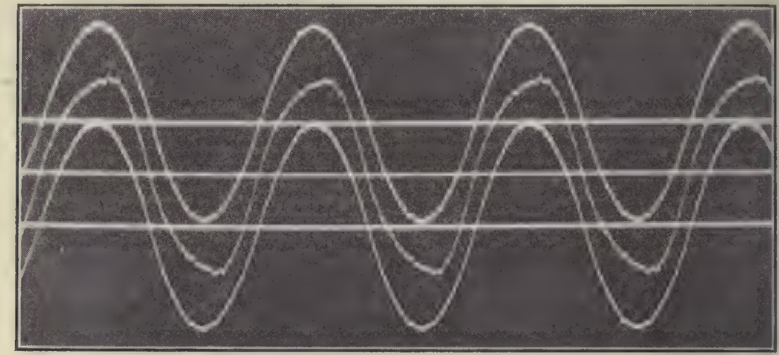
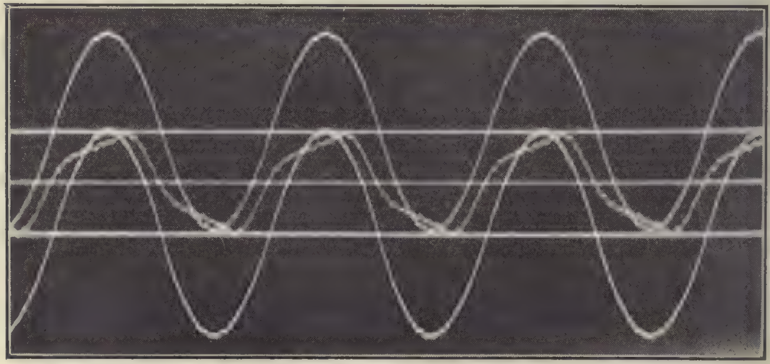


Fig. 3.—Oscillograph connections to obtain wave form of exciting current.

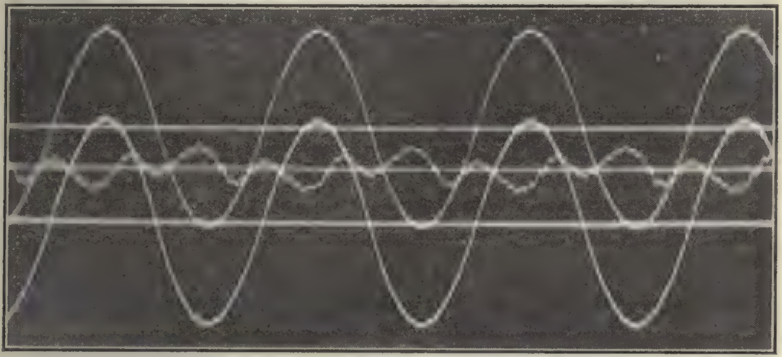


Oscillogram No. 17.

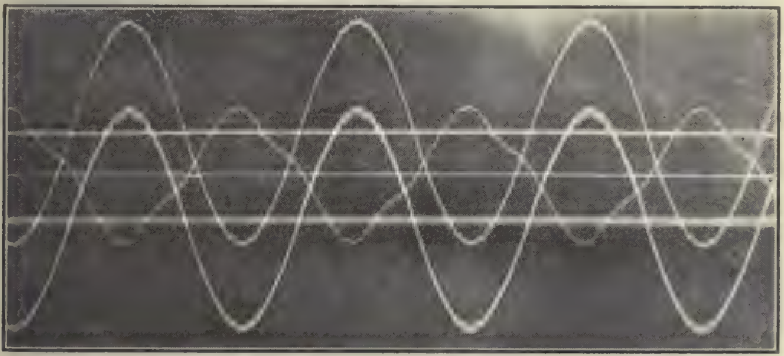


Oscillogram No. 18.

current of the transformer. In Osc. No. 18, the number of secondary turns has been reduced by one. There is thus subtracted from the difference-current a component in phase with and of the same wave form as the secondary current. Since the secondary current is approximately a sine wave the difference-current has the same harmonics in it as before, but these harmonics form a greater proportion of the total current and are therefore more noticeable. In Osc. No. 19 the compensation is two turns and it so happens that this almost exactly neutralizes the fundamental in the difference-current, leaving only the harmonics of the exciting current. In Osc. No. 20 the compensation has been carried to three turns, resulting in a reversal of phase of the difference-current and a dilution of the harmonics.

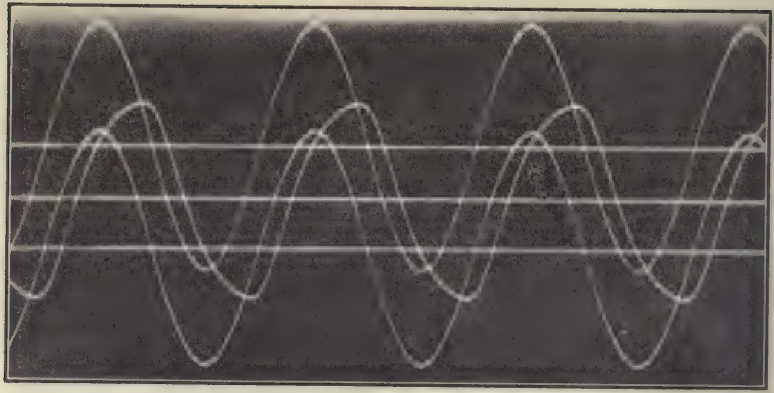


Oscillogram No. 19.

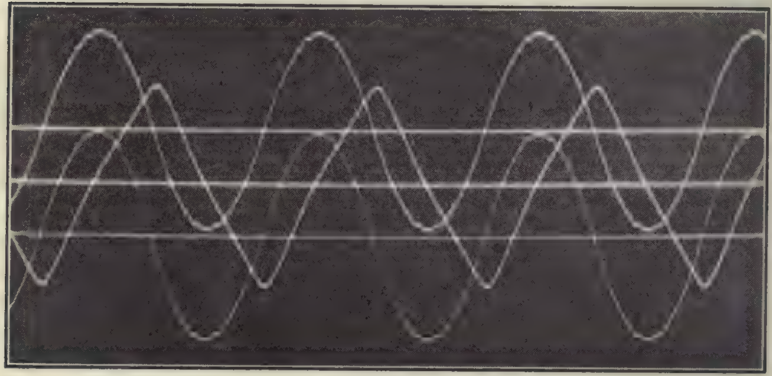


Oscillogram No. 20.

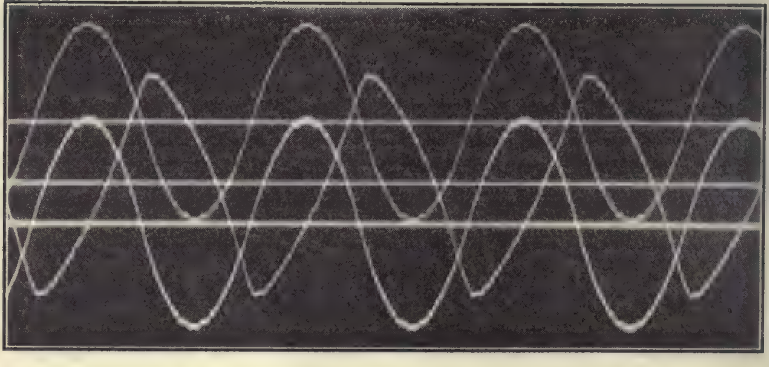
In Osc. Nos. 21 to 24 the total impedance in the secondary circuit was 1.18 ohms non-inductive resistance, and the density was 9600 lines per square inch. The high power factor of the secondary circuit causes the exciting current to lag with regard to the primary and secondary



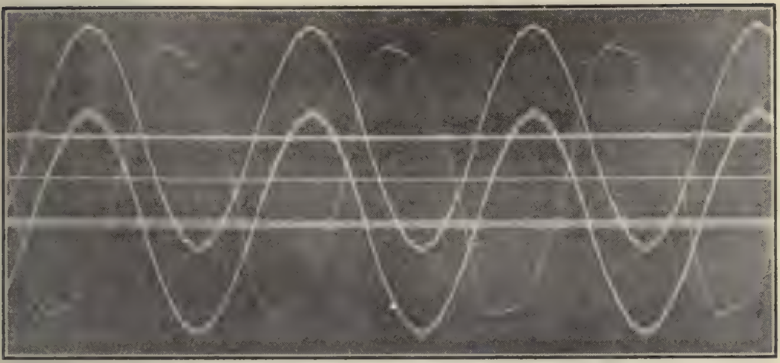
Oscillogram No. 21.



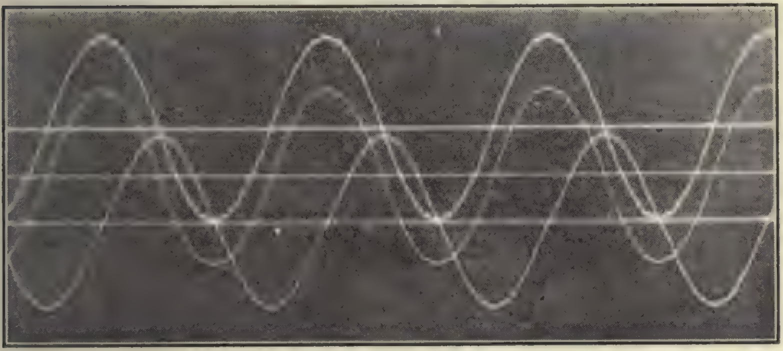
Oscillogram No. 22.



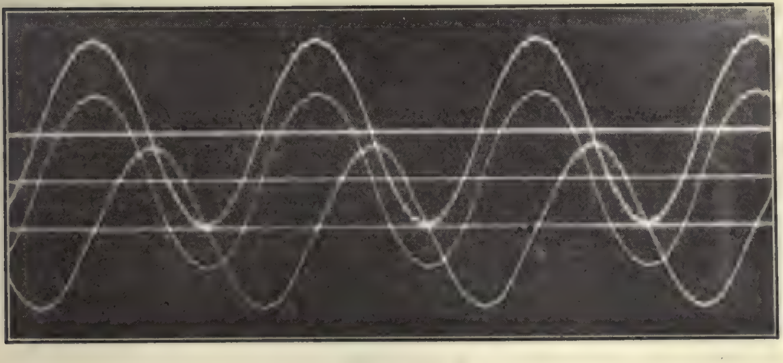
Oscillogram No. 23.



Oscillogram No. 24.



Oscillogram No. 28.



Oscillogram No. 29.

currents. In Osc. No. 21 there is no compensation, while in Osc. Nos. 22, 23 and 24 the number of secondary turns is reduced by one, two and three turns respectively with effects on the difference-current as shown in the oscillograms.

As before stated the readings of the wattmeter carrying this difference-current will depend upon the wave form of the voltage applied to it. The voltage V_p will be of the same wave form as I_p, which will be the same as that of the alternator voltage. Voltage V_Q is obtained from the secondary of a phase-shifting transformer, the wave form of which is shown in Osc. Nos. 28 and 29, the first wave in each case being that of primary voltage, and the other two those of two secondary voltages which are nominally in quadrature. In Osc. No. 28 the primary windings of the phase shifter were connected in Y while in Osc. No. 29 they were in delta. There is no noticeable difference between the wave forms of the primary and secondary voltages.

Since the wattmeter is excited by a voltage of approximately sine form the readings will be proportional to the fundamental components of the difference-current, the harmonics being neglected. Thus in the case of Osc. No. 19 the wattmeter reading would be practically zero in both phases, indicating correct ratio and phase angle although there really exists a difference of about 0.01 ampere between the primary and secondary currents as shown by the second wave on the oscillogram. Therefore the results obtained by this method are apparently in error as to ratio, but not as to phase angle, which is taken to mean the phase angle between the fundamental harmonics of primary and secondary currents. The error in the ratio measurement depends upon the value of the higher harmonics in the exciting current but is usually very small. Assuming in a certain case that the effective value of the harmonics is 25 per cent. of the effective value of the exciting current, and that the exciting current is 1.8 per cent. of the load current, the harmonics will constitute 0.45 per cent. of the load current. If conditions are such that with 5 amperes secondary current the difference-current is composed only of harmonics as in Osc. No. 19, the wattmeter will read zero and indicate a ratio of unity. Actually there is a difference-current of $\frac{0.45}{100}\times 5 = 0.0225$ ampere, and thus the true primary current is $\sqrt{5^2+0.0225^2} = 5.00005$ and the true ratio is 1.00001. The error in this case, due to neglect of harmonics by the wattmeter, is only 0.001 per cent., which is quite negligible. Also, as has been shown in the oscillograms, the percentage of harmonics in the exciting current of current transformers at their normal densities is not large.

Most of the chief precision methods of calibrating current transformers are subject to the same errors from differences between primary and secondary current wave form, including null methods involving balancing of shunt drops or mutual inductances whether the indicator of balance is a vibration galvanometer, separately excited dynamometer, or permanent-magnet galvanometer with mechanical rectifier. The same applies to deflection methods with separately excited instruments. An exceptional case is the method involving the use of two dynamometers for the separate measurement of primary and secondary currents. Therefore the errors due to the exciting current wave form in the method of measuring ratio and phase angle in this research are no greater than those due to the same cause in most of the best known precision methods, and are usually negligibly small.

THROUGH-TYPE PORTABLE CURRENT TRANSFORMERS

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Professor of Electrical Engineering
C. Kent Duff,
Research Assistant

It is generally known that the ratio and phase-angle errors of through-type portable current transformers are affected by changes in the position of the primary, but little definite information is available on this subject. The following results obtained from ratio and phase-angle measurements on several different transformers of well-known manufacture may be of interest.

Through-type portable current transformers are extensively used in making field tests on alternating-current apparatus where the current reaches values in excess of 200 amperes. The nominal ratio can be varied over a considerable range by using different numbers of turns in the primary which consists of the conductor of the circuit being measured, passed usually 1, 2, or 4 times through the hole in the case to give ratios in proportion 4:2:1.

The effect of various changes in the position of the primary depends very much upon the construction of the current transformer, especially as regards the arrangement of the secondary winding on the core. The three most usual types of construction are:—

Type 1.—Circular core with uniformly distributed secondary.

Type 2.—Rectangular core with secondary in the form of two coils on opposite sides of the core.

Type 3.—Rectangular core with a single secondary coil on one side of the core.

These types have been investigated experimentally.

Type 1.—In this type of current transformer the phase-angle error is practically independent of primary position, while the ratio error is only slightly affected by changes in the position of the primary, less than 0.1 of one per cent. variation having been observed in the measurements made.

As to ratio, the secondary current is least, for a given primary current and secondary burden, when the primary conductor passes in a straight line axially through the transformer; or an equivalent arrangement when the primary consists of four or more turns uniformly distributed, that is, with loops equally spaced around the outside of the case. All other arrangements of the primary give a greater secondary current.

The maximum secondary current for given primary current is obtained when all primary turns are looped as closely as possible around one side of the transformer.

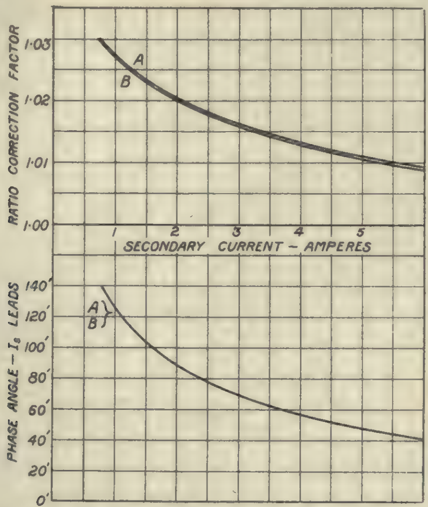

Fig. 1.—Ratio and phase-angle curves of type 1 current transformer; 400 to 5 amperes. *A*, straight axial primary; *B*, primary looped around one side.

Fig. 1 shows the ratio and phase-angle curves of a 400 to 5 ampere transformer with rated secondary burden of 10 volt-amperes, 80 per cent. power factor, at 60 cycles.

Ratio correction factor is the factor by which the marked ratio must be multiplied to give the true ratio. If it be greater than 1.0, the secondary current is too small, and *vice-versa*.

Curve *A* is for a straight axial primary; or its equivalent, a uniformly distributed primary of four or more turns. Curve *B* is the ratio curve when the primary is a coil one foot in diameter placed around one side of the transformer as in Fig. 2. The maximum difference is about 0.05 of one per cent. in the ratio. A primary coil of smaller diameter will give a greater secondary current, but one foot is about the minimum diameter practicable for stiff cables used in practice. All other positions of the primary give results between these two curves.

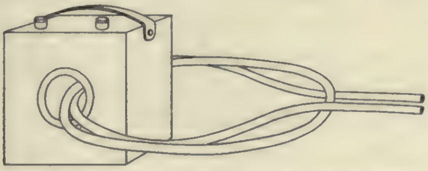

Fig. 2.—Type 1 current transformer; circular core.

Another transformer of the same type, of ratio 600 to 5 amperes, gives the curves shown in Fig. 3 with the same frequency and burden as above. Curve *A* is for a straight axial primary passing through the centre of the hole, or equivalent distributed primary. Curve *B* shows the result of moving the straight primary conductor close to one side of the hole. Curve *C* is obtained with the primary in the form of a coil one foot in diameter around one side of the core.

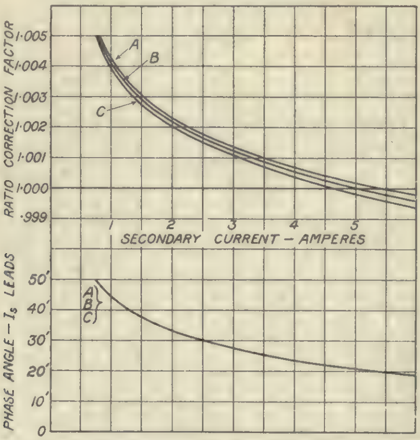

Fig. 3.—Curves for type 1 current transformer; 600 to 5 amperes. *A*, straight axial primary; *B*, straight primary close to one side of hole; *C*, primary looped around one side of core.

The phase-angle curves *A* and *B* of Fig. 1 are shown as identical, which means that the observed difference was definitely less than half a minute. The same is true for the phase-angle curves *A*, *B*, *C* of Fig. 3.

In endeavouring to obtain minimum errors of transformation in a transformer of Type 1 by suitable arrangement of the primary, ratio error only need be considered since phase angle is practically unaffected by changes in primary position. For the transformers investigated the minimum errors for the particular burden and frequency used are obtained with the primary as in Fig. 2. This may not always be the case, however, because most transformers are compensated, which means that the ratio of secondary to primary turns is less than the marked ratio of primary to secondary currents. It is, therefore, possible to have the true ratio curve lie partially or even totally below the line of marked ratio. The object of compensation is to make the ratio curve cross the line at about 4 amperes secondary current under average conditions of frequency and secondary burden so as to minimize the average ratio error. However, it is not always possible to attain this object perfectly, and thus it may be that the ratio curve lies mostly below the line. If compensation be the best for average conditions the ratio curve will be made too low by lower secondary burden, or higher power factor of secondary burden, or higher frequency. In such cases it may be best to use a straight axial or uniformly distributed primary.

It is evident, therefore, that in the general case of a compensated current transformer of Type 1, unless the approximate position of the ratio curve is known for the particular frequency and burden used, it is impossible to say how the primary should be placed for minimum average ratio error.

Type 2.—In this type of current transformer both ratio and phase angle are affected by variations in the position of the primary, observed variations in ratio in the investigations here discussed being about 0.1 of one per cent., and in phase angle about 2 minutes.

Regarding phase angle, any change in the position of the primary which increases the secondary leakage reactance will reduce the angle of lead of the secondary current with regard to the primary current. The angle of lead decreases as the primary is successively placed in the following positions:—

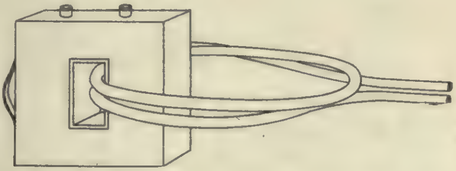

Fig. 4*a*.—Type 2 current transformer; primary in position 1.

1. Looped around one secondary coil, Fig. 4*a*.
2. Equally divided around the two secondary coils.
3. A straight axial conductor, or its equivalent of four or more turns uniformly distributed.
4. Looped around one end of the core between secondary coils.
5. Equally divided around the two ends of the core, Fig. 4*b*.

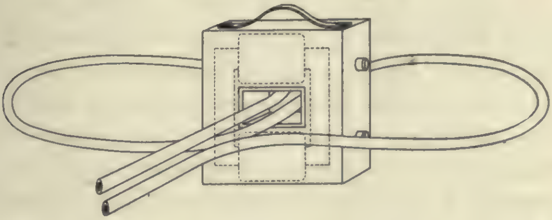

Fig. 4*b*.—Type 2 current transformer; primary in position 5.

The first and last of these positions produce the two extremes of phase-angle error, and these are shown in Fig. 5, curves *A* and *B* respectively, for a transformer of ratio 1,600 to 5 amperes. The frequency was 60 cycles and the secondary burden 25 volt-amperes at 80 per cent. power factor. The diameter of the primary loop was 14 inches, about the minimum for actual conductors in practice. A larger primary loop produces smaller variations. The maximum variation here is less than 2 minutes.

The ratio error is also affected by primary position, the secondary current decreasing as the primary is successively placed in the positions listed above. The ratio curve *A* is obtained with the primary as in Fig. 4*a*. Curve *B* shows the results with primary as in 4*b*. The maximum difference in the ratio error is 0.09 of one per cent.

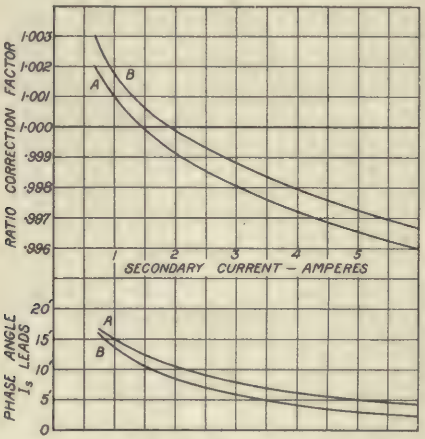

Fig. 5.—Curves for type 2 current transformer. *A*, primary as in Fig. 4*a*; *B*, primary as in Fig. 4*b*.

As to the best position of the primary to produce minimum errors, both ratio and phase-angle errors must be considered. In the above transformer, owing to over-compensation for the particular frequency and burden used, curve *B* is best for both ratio and phase angle, that is, the primary should be placed as in Fig. 4*b*. However, if the ratio curves had been above the line of marked ratio the position for least phase-angle error would have involved also the greatest ratio error, and *vice-versa*. It would then have been necessary to decide whether the primary coil position should be arranged to favour approximately 0.09 of one per cent. reduction in ratio error at the expense of 2 minutes' increase in phase angle, or *vice-versa*. It is worth noticing also that in power measurement 2 minutes in phase angle may, depending on load power factor, be a much more important change if the current transformer has a large phase angle than if the phase angle is small.

The decision depends upon the kind of measurements being made. Ammeter indications are affected only by ratio. Power-factor meters are affected only by phase angle. Wattmeter indications are dependent on both quantities, but phase-angle errors are more important at low power factors and ratio errors at high power factors.

If the phase-angle curves had been below the zero line, that is if the secondary current had been lagging instead of leading with regard to the primary current, the condition for least errors would again be changed. Conditions tending to make the secondary current lead less (or lag more) are lower power factor of secondary burden, or higher frequency, or any other thing which will reduce the power factor of the total secondary circuit. For the same reason a transformer of high ratio is more likely to have a secondary current lagging with regard to primary current than one of low ratio because it has more secondary turns and hence higher internal reactance and lower power factor.

If the power factor of the total secondary circuit is equal to that of the core the phase error will be zero, and if the secondary power factor is less than this the secondary current will lag behind the exciting current and therefore behind the primary current. See Fig. 6.

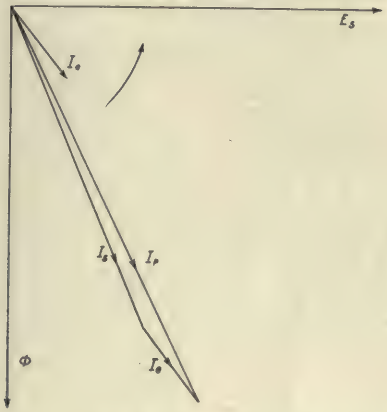

Fig. 6.—Vector diagram showing how secondary current may lag behind primary current when power factor of secondary circuit is low.

It is evident, therefore, that to know how to place the primary to minimize the errors, it is necessary to know whether the ratio and phase angle curves lie above or below the zero line.

Type 3.—Both ratio and phase-angle errors in this type of current transformer are greatly affected by changes in the position of the primary, observed variations in the transformer tested being 40 minutes in phase angle and 0.5 of one per cent. in ratio at rated secondary current and approximately rated burden, and greater variations at smaller currents.

As to phase angle, the secondary current is made to lead less (or lag more) with regard to the primary current by any change in primary position which tends to increase the secondary leakage reactance. The latter is minimum when the primary is wound as closely as possible over the secondary coil, and maximum when wound as closely as possible on the opposite side of the transformer core. The angle of lead of the secondary current with regard to the primary current decreases as the primary is successively placed in the following positions:—

1. Wound closely over the case on the same side as the secondary coil.
2. A coil 11 inches in diameter around the secondary coil, Fig. 9.
3. A straight axial conductor or its equivalent of four or more turns uniformly distributed around the core.
4. Two 11-inch coils around the two ends of the core.
5. A coil 11 inches in diameter around the side opposite the secondary coil.
6. Wound closely over the case on the side opposite the secondary coil.

Regarding ratio, increase of leakage reactance increases the total secondary impedance and hence ratio also. Therefore the ratio increases as the primary is successively placed in the six positions listed above.

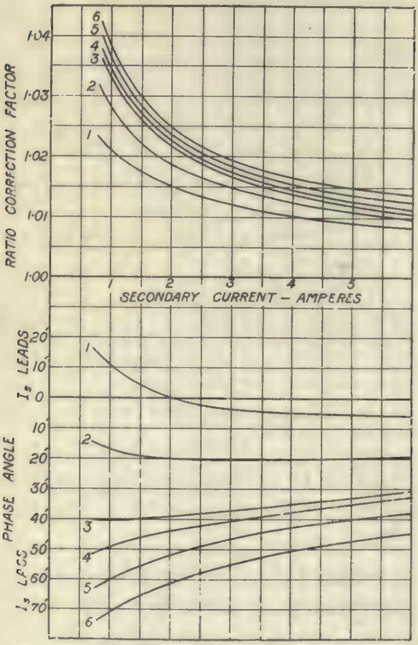

Fig. 7.—Curves for type 3 current transformer at 60 cycles; primary in positions 1 to 6.

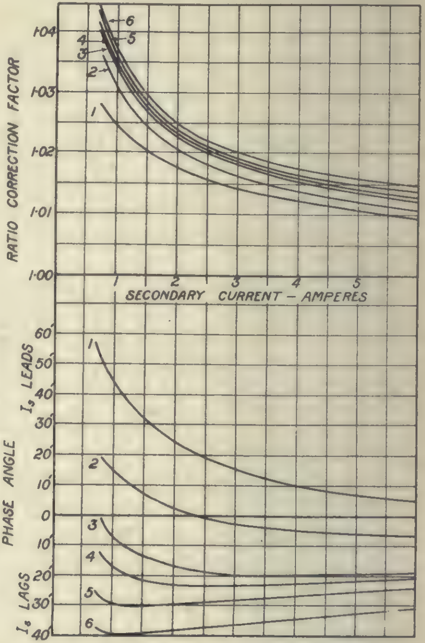

Fig. 8.—Curves for type 3 current transformer at 25 cycles; primary in position 1 to 6.

The ratio and phase-angle curves for a 1,600 to 5 ampere transformer with secondary burden of 11.2 volt-amperes at 90 per cent. power factor, 60 cycles, are given in Fig. 7 for the primary in the six positions listed above, curves 1 to 6 corresponding to positions 1 to 6. The corresponding curves for the same transformer and the same burden at 25 cycles are given in Fig. 8 for the same six positions of the primary.

The secondary leakage reactance in this transformer with primary in position 2 was about 3 ohms at 60 cycles, and with primary in position 5 about 7 ohms. These values are much larger than the total resistance in the secondary circuit, and the result is very low power factor of the total secondary circuit, which accounts for the secondary current lagging behind the primary current. For the same reason it is evident that the position of the primary has a greater influence on the characteristics of the transformer than a considerable change in the secondary burden because it produces a greater effect on the total impedance of the secondary circuit. These effects would be still more marked in a transformer of higher ratio, but less pronounced in one of lower ratio.

A transformer of this type in which the secondary winding is highly inductive is quite sensitive to frequency variations even with a non-inductive burden, but the effect of changes in primary position quite buries the effect of possible variation in frequency on a commercial system. Even a change in frequency from 60 to 25 cycles with constant primary position produces a smaller change in both ratio and phase-angle errors than does a change in primary from position 5 to position 2 with constant frequency. By comparing, for example, the phase-angle curve 5 in Fig. 7 with the phase-angle curve 5 in Fig. 8 the effect of a change in frequency will be seen. In contrast to this, by comparing phase-angle curves 5 and 2 in Fig. 7 the effect of a change in primary position will be seen. Similar relations exist in the corresponding ratio curves. It is interesting to observe that the variations in phase angle are approximately the same at the two frequencies, which is not true of the ratio errors.

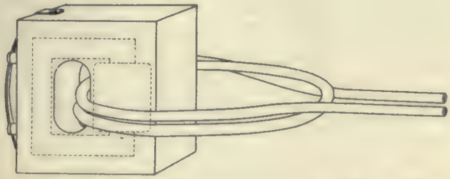

Fig. 9.—Type 3 current transformer; primary in position 2.

No general statement can be made as to the best arrangement of the primary for minimum errors in transformers of Type 3. In regard to the transformer tested, the best arrangement for average conditions would be position 2, that is, primary looped around the secondary coil as in Fig. 9. Stiff cable used in practice would probably not be bent smaller than 11 inches diameter which would give the ratio and phase angle curves marked 2 in Figs. 7 and 8. The transformer tested was not compensated, probably due to an oversight in construction. With proper compensation and properly disposed primary winding this type of transformer can be made to have reasonably small errors, but an indiscriminate arrangement of the primary may cause serious differences between actual and supposed transformation.

Summary—1. Ratio and phase-angle errors in through-type portable current transformers depend upon the arrangement of the primary winding. The nature and extent of the effect of changes in primary position depend upon the arrangement of the secondary winding on the core.

2. Transformers with circular cores and uniformly distributed secondaries are very little affected by changes in primary position, phase angle being practically independent and ratio varying less than 0.1 of one per cent. in the two transformers tested.

3. Transformers with rectangular cores and secondaries in two coils on opposite sides of the core are affected in both ratio and phase-angle errors by change in primary position. These changes were found to produce variations in phase angle of 2 minutes and in ratio about 0.1 of one per cent. in the transformer tested.

4. Transformers with rectangular cores and single secondary coils on one side of the core are very much dependent on primary position; ratio varying about 1 per cent., and phase angle about 1 degree in the transformer tested.

5. The position of the primary for minimum errors in measurements cannot be stated in general terms. It depends upon the type of transformer used, its compensation, the frequency, and the properties of the secondary burden. In addition, the kind of measurement must be considered since current measurement depends only on ratio, power factor by power-factor meters only on phase angle, while power measurement is affected by both quantities; ratio being more important at high power factors and phase angle at low power factors of load.

DETERMINATION OF TURN-RATIO OF CURRENT TRANSFORMERS

C. KENT DUFF,
Research Assistant

A simple method of finding the turn-ratio of current transformers without the use of auxiliary exploring-coils has been used in connection with investigations in the School of Engineering Research, University of Toronto. Nothing on this subject has hitherto been published as far as the writer is aware. Such a method and the resulting data might have several useful applications, two of which are here suggested.

(1) Manufacturers of current transformers would be able to check the turn-ratio of their product and detect any errors in compensation due to a mistake in the number of turns in either winding. This is mentioned especially because among a small number of transformers investigated two were found in which it was quite clear that errors had been made in the number of secondary turns, one being uncompensated and the other about 1 per cent. over-compensated.

(2) In certain methods of testing current transformers a knowledge of turn-ratio is required. See for example *Transmission Line Relay Protection*, by Woodrow, Roper, Traver, and McGahan, Proc. A.I.E.E., June 1919, p. 637, and *A Study of the Current Transformer With Particular Reference to Iron Loss*, by P. J. Agnew, Bulletin, Bureau of Standards, Vol. 7, No. 3, p. 431. The manufacturer's data on turn-ratio, even if obtainable, is not always correct as already noted.

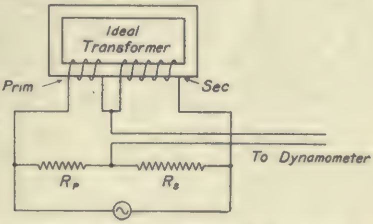

Fig. 1.—Simple bridge method of determining turn ratio of ideal transformer.

The general principle of the method is that in an ideal transformer without resistance or core reluctance an alternating voltage applied to the two windings connected in series in the same sense will be divided between them in the ratio of the number of turns in each. The turn-ratio might then be determined by a potentiometer or bridge balance method as suggested in Fig. 1. In actual transformers there are two conditions which modify the application of the above principle. These are copper resistance of the windings and leakage flux, by which is meant flux that does not completely interlink both windings and will, therefore, cause a greater average voltage drop per turn in one winding than in the other The resistance drops in the two windings, if not proportional to the number of turns in each, will cause a greater average voltage per turn in one winding than in the other and will, therefore, give slightly erroneous results. These conditions necessitate certain precautions the details of which are explained below.

The resistances of the two windings are made approximately proportional to turns by adding the necessary compensating resistance externally to one winding, usually the primary. If r_p and r_s are the primary and secondary resistances respectively and n the marked ratio of primary to secondary currents, the necessary extra primary resistance is $\frac{r_s}{n} - r_p$. If this gives a negative result it means that extra resistance is required in the secondary of value $(r_p \times n) - r_s$. Strictly n should be the turn-ratio and not the marked ratio, but the former is supposed to be unknown and in most cases will not differ from the latter more than 2 or 3 per cent. Also, the resistance drop in the windings will usually be quite small compared to the reactance drop since both current and resistance are small. The leakage flux is in most cases a more serious source of error than the resistance of the windings. Air forms part or all of the path of such flux, and the greater part of the reluctance of the leakage paths will, therefore, be in air which will cause the leakage flux to be proportional to and in phase with the current in the windings, while the core flux is approximately proportional to, and in quadrature with, the impressed voltage. These facts suggest that error may be avoided by using as an indicator of balance a sensitive dynamometer or A.C. galvanometer with its field separately excited in phase with the current in the transformer. The dynamometer will then be insensible

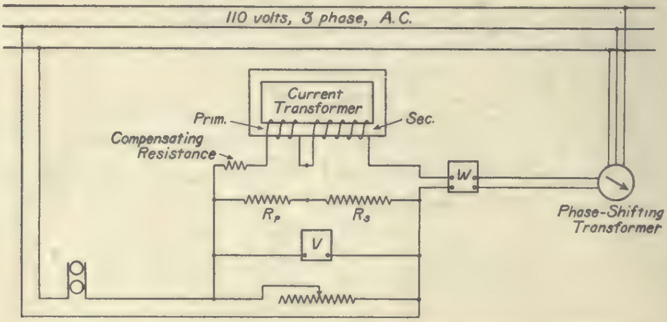

Fig. 2.—Diagram of connections to obtain voltage in quadrature to current through transformer.

to all voltages in quadrature with the current, that is, to voltages induced by the leakage flux.

In order to excite the dynamometer in phase with the transformer current a preliminary test is made in which a phase-shifting transformer as shown in Fig. 2, is adjusted to give a secondary voltage in quadrature with the current-transformer current. The dynamometer field current, adjusted in quadrature with the phase-shifter voltage, will then be in phase with the current in the current transformer. Observations are, therefore, made of the phase-shifter settings required to make the wattmeter *W* indicate zero for each of a number of voltages applied to the current transformer. If a phase-shifting transformer is not available the potentiometer scheme suggested in Fig. 3*a* may be used on three-phase circuits. The phase across which the potentiometer should be connected depends upon the phase rotation of the system and on the power factor of the current-transformer core, but is easily determined by trial. A similar scheme may be used on two-phase circuits. On single-phase circuits a condenser and resistor in series are connected across the line and the wattmeter is connected across the resistor as shown in Fig. 3*b*. A 100 ohm rheostat and 25 telephone condensers of 2 microfarads each give the required range of phase displacement at 60 cycles. The ratio of R_s to R_p should be made approximately equal to the expected turn-ratio so that subsequent alterations will be small.

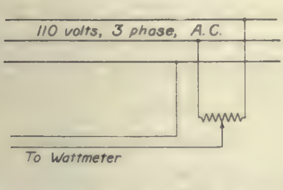

Fig. 3*a*.—Alternative method of obtaining quadrature voltage.

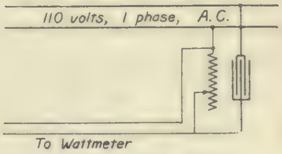

Fig. 3*b*.—Method of obtaining quadrature voltage from single phase circuit.

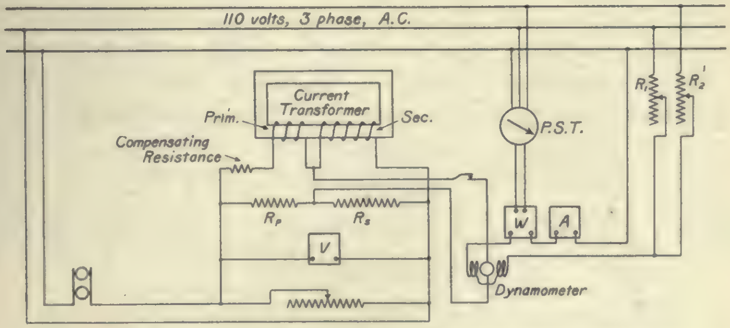

Fig. 4.—Connections for determining turn-ratio of current transformer.

The wattmeter is then removed from the current-transformer circuit and placed in the dynamometer field circuit and other connections are made as indicated in Fig. 4. With the same combinations of voltage and phase-shifter settings as found in the preliminary test, the dynamometer field current is adjusted to make $W=0$ by suitably proportioning the two resistances R_1 and R_2. On a single-phase circuit a combination of a variable reactor and resistor in series with the dynamometer field will effect the same result. The dynamometer field current is thus placed in quadrature with the phase-shifter voltage, which is itself in quadrature with the transformer current. Hence the dynamometer is excited in phase with the transformer current as required by the conditions. The non-inductive resistances R_s and R_p are then adjusted to make the dynamometer indicate zero. The ratio of R_s to R_p is then the turn-ratio of the transformer. The true values of the resistances R_s and R_p should be measured as accurately as possible for close results. Results should be the same for any applied voltage provided always that the dynamometer is excited in phase with the transformer current. A voltage which gives the maximum core power factor as indicated by the phase-shifter settings is a very good voltage to use, though somewhat higher voltages may also be used to advantage providing saturation is not too closely approached. The highest available commercial frequency is most suitable because it permits of higher transformer voltages with which closer balance can be obtained. A frequency of 60 cycles was used in obtaining the following results on two different types of current transformer. The dynamometer used gave a perceptible deflection (½ mm. at 1 metre radius) with 0.0003 volt on the moving coil and full field excitation of 2 amperes in phase with the voltage applied to the moving coil.

A 15,000 volt switchboard current transformer of marked ratio 150 and 75 to 5 amperes was tested. It was known to have 296 secondary turns and a total of 20 primary turns, or a true turn-ratio of 14.8. The primary resistance was 0.0085 ohm with sections connected in series and the secondary resistance was 0.455 ohm. Therefore extra primary resistance required to make the ratio of IR drops equal to the marked ratio of 75 to 5 was $\frac{0.455}{15}-0.0085=0.0219$ ohm. Phase-shifter settings (given as read on an arbitrary scale on which 90 degrees represents electrical quadrature) are included in the following results in which $\frac{R_s}{R_p}$ is the measured turn-ratio.

Impressed volts on C.T.	Phase-shifter settings	R_s	R_p	$\frac{R_s}{R_p}$	Per cent. error in turn-ratio measurement
75	68°	3005.9	203.08	14.8016	0.01 of 1 per cent.
60	$69\frac{1}{2}$°	3005.9	203.08	14.8016	0.01 of 1 per cent.
45	$72\frac{1}{2}$°	3005.9	203.08	14.8016	0.01 of 1 per cent.

It is not intended to claim the high accuracy in the measurement of R_s and R_p, which the number of significant figures used might seem to imply. However, extreme care was taken in arriving at the above values in order to determine whether there are any errors inherent in the method itself or whether the accuracy is limited only by the errors involved in the practical details of execution. The resistances given are the average of a number of measurements made on an unusually high grade certified bridge. In the determination of the ratio of two resistances, relative accuracy is the only essential and the relative accuracy of the bridge used is even better than its absolute accuracy. The order of magnitude of the possible error in measurement of the relative values of R_s and R_p would be about the same as the actual difference between the measured value and the true value of the turn-ratio, namely, 0.01 of 1 per cent.

A through-type portable current transformer of 1,600 to 5 amperes marked ratio was also tested. It had 320 secondary turns, which means that it was uncompensated though probably not so intended by the maker. In order to give a turn-ratio of 10 to 1, 32 turns were wound on as primary and the following results were obtained.

Impressed volts on C.T.	Phase-shifter settings	R_s	R_p	$\frac{R_s}{R_p}$	Per cent. error in turn-ratio measurement
60	54°	4999.6	500.16	9.996	0.04 of 1 per cent.
40	$56\frac{1}{2}$°	4999.6	500.16	9.996	0.04 of 1 per cent.
25	$61\frac{1}{2}$°	4999.6	500.16	9.996	0.04 of 1 per cent.

In both these cases the error in the determined turn-ratio is no greater than that which is to be expected due to the measurement of the resistances and hence we may reasonably conclude that any other errors inherent in the method are negligible. In most practical cases the limit of accuracy will be determined by the accuracy in resistance measurement, but in high ratio transformers, especially those with single-turn primary, the limiting factor may prove to be the sensibility of the dynamometer.

MINIMIZING THE ERRORS OF CURRENT TRANSFORMERS BY MEANS OF SHUNTS

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In the measurement of large quantities of energy for which correspondingly large charges are made a small percentage error in the current transformers operating the meters may involve quite a considerable amount of money. The maximum possible accuracy is, therefore, worth while in current transformers used for this class of service.

Current transformers are usually "compensated" for ratio by adjusting the number of secondary turns. This type of compensation will be termed "turns-compensation" to distinguish it from the "shunt-compensation" herein discussed.

Turns-compensation is accomplished by making the ratio of secondary to primary turns slightly less than the nominal ratio of primary to secondary currents. Since the secondary ampere-turns are equal to the vector difference between the primary ampere-turns and the exciting ampere-turns, the true ratio of primary to secondary currents is always greater than the ratio of secondary to primary turns. Turns-compensation, therefore, reduces the ratio error, but the correction is only an approximate one for average conditions of use. When it is considered that one current transformer is usually intended to serve for all frequencies from 25 to 133 cycles, and for a rated secondary burden of unstated power factor, it is apparent that under the particular conditions of frequency and burden in a given case, there will generally be room for further improvement in the accuracy of the ratio. The phase-angle error cannot be reduced by turns-compensation.

In any particular case it is usually possible to reduce both the ratio and phase-angle errors of a current transformer by suitable turns-compensation combined with the use of a non-inductive or a condensive shunt connected across either the primary or secondary terminals.

Primary shunts have advantages from another point of view, in that they tend to protect the current transformer and secondary circuit from the effect of high-frequency disturbances. Secondary shunts tend to protect only the secondary instruments in case high frequency currents are transmitted to the secondary. Condensive shunts are more effective for these purposes than non-inductive shunts since the impedance of the former to high-frequency currents is much less than their impedance at commercial frequencies, while the impedance of non-inductive shunts is independent of frequency. The protective action of the latter, therefore, depends upon the existence of a certain amount of inductance in the circuits across which they are connected.

TURNS-COMPENSATION

It has been stated that the secondary ampere-turns are equal to the vector difference between the primary ampere-turns and the exciting ampere-turns, or in symbols,

$$I_s n_s = I_p n_p - I_e n_p \text{ vectorially} = n_p (I_p - I_e)$$

Rearranging,

$$\frac{I_p - I_e}{I_s} = \frac{n_s}{n_p}$$

If there is no turns-compensation $\frac{n_s}{n_p}$ is the nominal ratio. The true ratio, $\frac{I_p}{I_s}$, is greater than $\frac{I_p - I_e}{I_s}$, therefore greater than $\frac{n_s}{n_p}$, therefore also greater than the nominal ratio. In other words, the secondary current is too low, the vector difference $I_p - I_e$ being usually numerically less than I_p. By reducing n_s to n_{s1}, that is by turns-compensation, the secondary current increases from I_s to I_{s1}. Assuming I_e to remain unchanged the secondary ampere-turns will be the same as before or $n_s I_s = n_{s1} I_{s1}$.

The new value of the true ratio would therefore be

$$\frac{I_p}{I_{s1}} = \frac{I_p}{I_s \times \frac{n_s}{n_{s1}}} = \frac{I_p}{I_s} \times \frac{n_{s1}}{n_s},$$

or, in other words, the true ratio would vary directly with the number of secondary turns, primary turns remaining constant. Such is not exactly true, however, because the increased secondary current requires proportionately increased secondary induced voltage, and the number of turns in which that voltage must be induced is smaller than before, hence the flux must increase as the square of the increase in current. This requires a greater exciting current than before, and therefore the percentage reduction in the ratio is somewhat less than the percentage reduction in secondary turns.

The ratio and phase-angle curves of a certain current transformer* with fixed burden and frequency and various compensations are shown in Fig. 1. The burden was 25 volt-amperes at 87½ per cent. power factor, the frequency was 60 cycles, and the nominal ratio was one to one, the number of primary turns being 224. The numbers of secondary turns were 224, 223, 222 and 221 in curves *A*, *B*, *C*, and *D* respectively.

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*The current transformer used in this and the following measurements was of shell type. The frequency throughout was 60 cycles. The burden (except for Fig. 5) was 25 v.a., at 5 amperes and 87½ per cent. power factor; that is, with $R_s = 0.875$ ohms and $X_s = 0.487$ ohms.

Table I shows the percentage reduction in ratio, which is seen to be always less than the percentage reduction in turns.

TABLE I

Change in secondary turns	Per cent. change in turns	Per cent. change in observed ratio at 1 ampere secondary current	Per cent. change in observed ratio at 5 amperes secondary current
224-223	0.448	0.400	0.430
223-222	0.450	0.402	0.432
222-221	0.452	0.404	0.434

The number of figures shown in the values of per cent. change in observed ratio is not intended to imply such extreme accuracy in ratio observations. The last figures are estimated for the purpose of comparison with the per cent. change in turns.

The effect of turns-compensation is, therefore, to lower the whole ratio curve by an amount which is slightly less than the percentage reduction in secondary turns, and at the same time to slightly increase the slope of the ratio curve. The phase angle is not noticeably affected by a small percentage change in secondary turns,

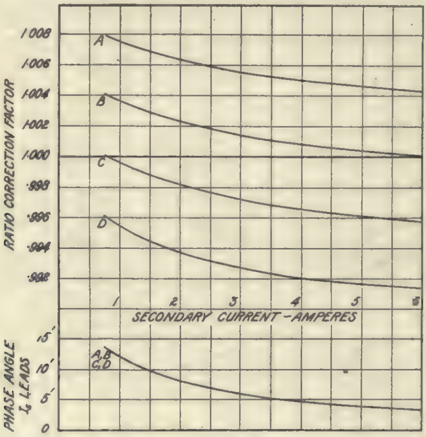

Fig. 1.—Ratio and phase-angle curves of a current transformer with four different turns-compensations. Primary turns, 224; Secondary turns: *A*, 224; *B*, 223; *C*, 222; *D*, 221.

In Fig. 1 the most favourable ratio curve would be either *B* or *C*. In most cases *B* would probably be preferred since its ratio error is less than that of *C* between 2 and 6 amperes secondary current. It would be still better if the ratio curve could be made to cross the line at about 4 amperes. However, that is not possible in this case by turns-compensation alone because the least change that can be made is one turn, which causes nearly 0.45 per cent. change in ratio.

SECONDARY NON-INDUCTIVE SHUNTS

The effect of a non-inductive shunt connected across the secondary terminals of a current transformer depends upon the characteristics of the secondary burden.

When the secondary burden is non-inductive the effect of a secondary non-inductive shunt will be to increase the ratio* without affecting the phase angle. If the resistance of the shunt is high compared to that of the secondary burden, the total impedance of the secondary circuit will not be appreciably affected by it, and the current in the secondary burden will be approximately $\frac{r_c}{r_c+R_s}$ times its former value, where R_s is the resistance of the secondary burden and r_c is the resistance of the non-inductive shunt. The new ratio will, therefore, be $\frac{r_c+R_s}{r_c}$ times its former value, or the percentage increase is $\frac{R_s}{r_c}\times 100$. The ratio curve can thus be raised by any desired amount according to the value of the resistance of the shunt, instead of by steps as in turns-compensation.

If the secondary burden is inductive the effect of a secondary non-inductive shunt will depend upon the magnitude and power factor of the burden. Consider first the effect in the case of an inductive burden of zero power factor if such were possible. The non-inductive shunt in this case would take a current in quadrature with the current in the burden, and if the resistance of the shunt were large enough to make its effect on the total secondary impedance negligible, the ratio would be unchanged, while the phase angle, considered positive when the secondary current leads with regard to the primary current, would be reduced by an angle $\tan^{-1}\frac{X_s}{r_c}$ where X_s is the reactance of the secondary burden. Since in most current transformers under operating conditions the secondary current leads, the secondary non-inductive shunt would thus reduce the phase-angle error.

In practice the secondary burden usually has a power factor between

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*In referring to current transformers provided with primary or secondary shunts, ratio is considered to mean the ratio of the primary line current to the current in the secondary burden, not the ratio of the currents in the primary and secondary windings.

50 and 100 per cent. depending on the frequency of the circuit and the type of instruments used. In such cases a secondary non-inductive shunt affects both ratio and phase angle, its effect on ratio being proportional to the resistance of the secondary burden and its effect on phase angle proportional to the reactance.

The ratio is increased approximately $\frac{R_s}{r_c} \times 100$ per cent. and the phase angle is reduced approximately by an angle $\tan^{-1} \frac{X_s}{r_c}$.

The effect of the shunt can be understood more readily by reference to the vector diagram, Fig. 2, in which I_s is the secondary current and I'_s its counterpart in the primary assuming a turn-ratio of one to one. V_s is the secondary terminal voltage leading I_s by an angle $\tan^{-1} \frac{X_s}{R_s}$, and I_c is the current in the non-inductive shunt which is in phase with V_s, and is represented in the primary by I'_c. E_s is the secondary induced voltage, and ϕ the flux producing it, while the exciting current I_e is required to maintain the flux. I_e and I_c are both exaggerated in this diagram for the sake of clearness. Without any secondary shunt the primary current would be represented by the broken line I'_p. With a shunt on the secondary the primary current must be equal to I_p to hold the same secondary current. The ratio $\frac{I_p}{I'_s}$ is greater than $\frac{I'_p}{I'_s}$, but the phase angle between I_p and I'_s is less than that between I'_p and I'_s. The current in the secondary shunt, or rather its counterpart in the primary, is seen to have the same effect as so much addition to the exciting current.

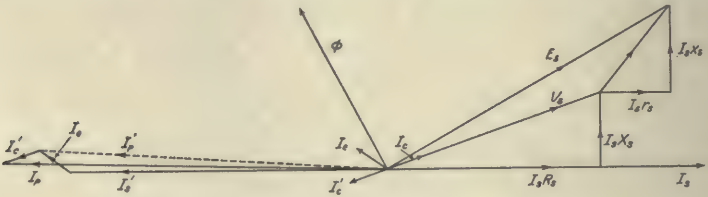

Fig. 2.—Vector diagram of current transformer with a non-inductive shunt in the secondary circuit.

In the case of the transformer before referred to, with 222 secondary turns and a burden of 25 volt-amperes at 87½ per cent. power factor, 60 cycles, using a secondary non-inductive shunt of 220 ohms, curves A, Fig. 3, were obtained. The improvement over curves B and C, which are the same as B and C in Fig. 1, is obvious. In this case R_s is 0.875.

X_s is 0.487, and r_c is 220 ohms. The calculated change in ratio is $\frac{R_s}{r_c}\times 100=\frac{0.875}{220}\times 100=0.4$ per cent.

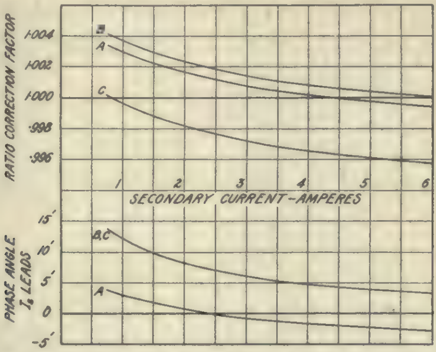

Fig. 3.—Curves showing effect of a non-inductive shunt in the secondary circuit. *A*, secondary turns 222, secondary non-inductive shunt of 220 ohms. *B* and *C*, same as in Fig. 1, (no sec. shunt).

Similarly the calculated change in phase angle is $\tan^{-1}\frac{X_s}{r_c}=\tan^{-1}\frac{0.487}{220}=\tan^{-1}0.0022=7\frac{2}{3}$ minutes. The observed changes at different secondary currents are shown in Table II. The differences between the calculated and observed changes is due to the effect of the shunt on the total secondary impedance, and perhaps also to small inaccuracies in the observations.

TABLE II

Secondary amperes	1	2	3	4	5	6	Average observed change	Calculated change
Per cent. ratio change	0.37	0.38	0.39	0.4	0.4	0.4	0.39	0.40
Phase-angle change in minutes	9'	7'	$6\frac{1}{3}$'	6'	6'	6'	$6\frac{3}{4}$'	$7\frac{2}{3}$'

Predetermination of the effect of a secondary non-inductive shunt requires only a knowledge of the resistance of the shunt and of the resistance and reactance of the secondary burden. Conversely the resistance of the secondary shunt required to effect desired changes in ratio or phase angle may be calculated if the constants of the secondary burden be known. For example, in Fig. 1, curve *C*, at 4 amperes secondary current the ratio is 0.375 per cent. low, and the phase angle 4 minutes high. The approximate shunt resistance required to make the ratio correct at 4 amperes is $\frac{0.875}{0.375}\times 100=234$ ohms. The approximate shunt resistance required to make the phase angle zero at 4 amperes is $\frac{X_s}{\tan 4'}=\frac{0.487}{0.0012}=410$ ohms. Since both ratio and phase angle cannot be made simultaneously zero at 4 amperes secondary current, it must be decided which error it is more important to eliminate or else a compromise has to be made

A secondary non-inductive shunt will not reduce the ratio error unless the turns-compensation is such that the ratio is too low. If ratio is too high it becomes necessary to reduce secondary turns if improvement is to be made by this type of shunt-compensation. The number of turns on the secondary of a transformer already built cannot readily be changed. However, it is sometimes possible to pass an extra turn or two through the core and insulate them sufficiently from the primary. If connected in series with the secondary and in the same sense on the core, the total number of secondary turns is increased; while, if opposing the secondary, the effect is practically the same as if an equal number of secondary turns were removed.

Non-inductive resistors suitable for use as secondary shunts may be easily and cheaply obtained in compact form from various manufacturers. The normal voltage that secondary shunts are required to stand would be from 10 to 15 volts, but the voltage might rise to several times this value for short periods as, for instance, when an increase in current is occasioned by a heavy overload or a short circuit on the system. A high voltage would also result from a break in the secondary, circuit of the current transformer, and the shunt would quickly be burnt out.

SECONDARY CONDENSIVE SHUNTS

A condenser connected across the secondary terminals of a current transformer in parallel with the burden will take a current which is proportional to the secondary terminal voltage and approximately in quadrature with it. Replacing I_c and I'_c in Fig. 2 with a leading current such as would be taken by a condenser, and following a similar course of reasoning to that given in regard to the secondary non-inductive shunt, the effects of a secondary condensive shunt will readily be seen. Its effect on ratio will be proportional to the reactance of the secondary burden, and its effect on phase angle proportional to the resistance, which are converse effects to those of a non-inductive shunt. Assuming that the condenser does not appreciably affect the total secondary impedance, that the voltage applied to it is a sine wave, and that its power factor is zero, it will *reduce* the ratio by an amount $X_s\, C\omega \times 100$ per cent., and lower the phase angle by an angle $\tan^{-1} R_s\, C\omega$, where

X_s and R_s are the constants of the secondary burden, C the capacity of the secondary condensive shunt in farads and ω is $2\pi f$ where f is the frequency of the circuit.

Actually there are several factors which make the above assumptions untrue so that the formulae are only approximate, though in many cases the results will hold with sufficient accuracy for practical purposes.

The power factor of a condenser is to all intents and purposes zero, its phase angle being very nearly 90 degrees. The impedance of the shunt may be of the order of magnitude of one per cent. of the impedance of the secondary burden which will cause a very small change in the total impedance of the secondary circuit. On the other hand the secondary voltage will scarcely ever have the assumed sine form and in some cases it may be something quite different. The wave form of the secondary voltage is influenced by at least three factors. If the primary current is not a sine wave it is practically certain that neither the secondary current nor secondary terminal voltage will be a sine wave. If the primary current is a sine wave the secondary current will not be quite free from harmonics because of the harmonics in the exciting current. If the secondary current is a sine wave the secondary terminal voltage will not be so unless the secondary circuit contains no iron core devices such as relays or induction meters which have cyclically varying impedances. The matter of harmonics and frequency variation is of more importance in condensive shunts than in non-inductive shunts which do not change their impedance with frequency. Primary current wave form may vary greatly, being much distorted under certain conditions, such as in a synchronous motor running light at unity power factor, while the same machine under load would take a current of much smoother form. Shunt-compensation, by means of a condenser, might be correct for the latter case, but it might cause serious errors in the former. However, the measurements made at light load would probably not be so important.

Paper telephone condensers in units of ½ to 2 microfarads are inexpensive, compact and otherwise well suited for use as condensive shunts. They are usually tested to 500 volts d.c., which is ample for any but very abnormal conditions, such as a severe short circuit on the system or an open secondary circuit which would cause voltages endangering the condenser insulation.

Fig. 4 gives the ratio and phase-angle curves of the current transformer already mentioned, with the same conditions of burden and frequency as in Fig. 1. Curves *B* and *C* are obtained with 223 and 222 secondary turns respectively, and no secondary shunt. Curve *A* results when a secondary condensive shunt of about 7 microfarads is used with 223 turns in the secondary. The calculated effect of a 7 microfarad

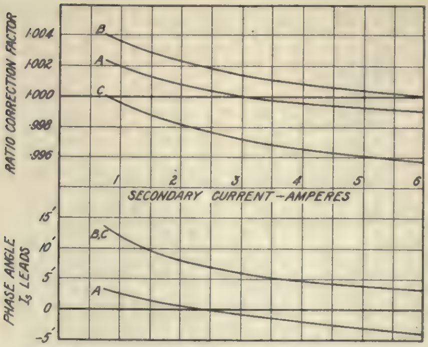

Fig. 4.—Curves showing effect of a condensive shunt in the secondary circuit. *A*, secondary turns 223, secondary condensive shunt of 7 m.f.d. *B* and *C*, same as in Fig. 1 (no sec. shunt).

shunt is to reduce the ratio $0.487 \times 7 \times 10^{-6} \times 377 \times 100$ per cent. or 0.13 per cent., and to lower the phase angle by $\tan^{-1} 0.875 \times 7 \times 10^{-6} \times 377 = \tan^{-1} 0.0023$ or 8 minutes. The observed changes, that is the differences between curves *A* and *B*, Fig. 4, are given in Table III.

TABLE III

Secondary amperes	1	2	3	4	5	6	Average observed change	Caclculated change
Per cent ratio change	0.10	0.11	0.12	0.12	0.12	0.12	0.115	0.13
Phase-angle change in minutes	9½′	7′	6½′	6½′	7′	7′	7.2′	8′

The secondary condensive shunt is useful when the original ratio is too high and the turns-compensation cannot be changed, since it reduces the ratio. The required capacity of a secondary condensive shunt to effect desired changes in ratio or phase angle may be calculated approximately from the formulae already given. It will generally be found that different values of capacity are required for the desired ratio and phase angle corrections. In such cases a compromise has to be made.

The above discussion has been on the application of condensive shunts to current transformers operating on a fixed burden. A secondary condensive shunt is advantageous also in current transformers not operating on a fixed burden, as, for instance, portable current transformers in which the secondary burden may be frequently changed. If the burden is small, in the limit zero, a secondary shunt has little or no effect since it is practically short-circuited. The ratio and phase angle would, therefore, be unchanged by the removal of the secondary shunt. Assuming the shunt to be removed, an increase of the secondary burden would tend to raise the ratio and to increase the phase angle, if the power factor of the secondary burden is greater than that of the secondary winding, as is usually the case with portable current transformers supplying meters. With a secondary condensive shunt the shunt tends to neutralize the above effects of increasing burden and hence to keep the ratio and phase angle curves nearly constant over a considerable range of secondary burdens and power factors. If these sets of nearly identical curves can be located near the line of zero errors, a very desirable condition is realized. In order to do this, the transformer must be so designed that the errors are a minimum with the secondary short-circuited. To keep the phase angle small under this condition the transformer must have considerable leakage reactance so that the power factor of the secondary winding will be low and about equal to that of the core. Turns-compensation may be employed to make the ratio as near correct as possible with secondary short-circuited. A ratio and phase angle test should then be made at rated frequency, using a burden which in magnitude and power factor represents the average burden which the transformer will be called upon to supply. From this data the required capacity can be calculated or the capacity can be determined experimentally during the ratio and phase angle test.

Such a current transformer equipped with a secondary condensive shunt, if designed for minimum errors at one given frequency, will not have such good characteristics at a different frequency. It is also subject to wave-form errors as before shown.

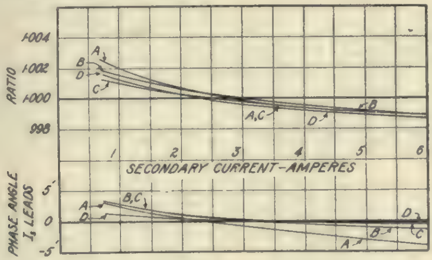

Fig. 5.—Curves showing effect of various burdens. Prim. turns 224; sec. turns 223; sec. shunt 7 m.f.d. Burden: *A*, 25 v.a. @ 87½ % p.f.; *B*, 16 v.a. @ 93½ % p.f.; *C*, 8.55 v.a. @ 93.9 % p.f.; *D*, sec. short-circuited.

An example of the results of this type of design is given in Fig. 5, which shows ratio and phase angle curves of the transformer of Fig. 1 with various burdens made up of random combinations of Weston dynamometer-type portable indicating ammeters and wattmeters. The frequency was 60 cycles and the capacity of the secondary shunt 7 microfarads.

Non-inductive shunts are not applicable to this type of service as they cause the ratio to vary more with change in burden than if no shunt were used.

COMBINATIONS OF SECONDARY NON-INDUCTIVE AND CONDENSIVE SHUNTS

As has been shown, it rarely occurs that a given shunt will make the desired corrections in both ratio and phase angle, though it may reduce both these errors. However, with the proper turns compensation, a combination of a non-inductive shunt with a condensive shunt connected in parallel across the secondary terminals will often make possible the desired changes in both errors. Equating the desired percentage *reduction* in ratio to $100\left(X_s C\omega - \frac{R_s}{r_c}\right)$ and the desired reduction in phase angle to $\tan^{-1}\left(R_s\, C\omega + \frac{X_s}{r_c}\right)$ and solving as simultaneous equations, values of C and r_c are obtained which denote approximately the required constants of the condensive and non-inductive shunts respectively. If the results are negative, or not real quantities, it means that it is not possible to completely neutralize both errors with the existing turns-compensation. If secondary turns can be changed, say from n_s to n_{s1}, recalculation should be made on the assumption of this change in secondary turns. Thus the desired percentage *reduction* in ratio is equated to $100\left\{(X_s\, C\omega - \frac{R_s}{r_c}) + \frac{(n_s - n_{s1})}{n_s}\right\}$. It may be necessary to make n_{s1} one or possibly two turns greater or less than n_s according to conditions. Conditions are sometimes such that no change in secondary turns will make a real solution possible. Only partial compensation can then be obtained.

SECONDARY INDUCTIVE SHUNTS

By reference to Fig. 2 it will be seen that a secondary inductive shunt, taking a current lagging with regard to V_s, will have little or no beneficial effect on either ratio or phase angle.

Some experiments were made in an attempt to use the saturation properties of an iron core inductive shunt in order to flatten out the ratio curve while employing turns-compensation to correct the ratio and non-inductive or condensive shunts to compensate for phase angle. The conclusion was reached that this is not practically feasible because, though part of the ratio curve may be flattened, its slope is increased in other parts, and ratio rises rapidly as the current increases above its rated value, due to saturation in the iron core impedance.

PRIMARY NON-INDUCTIVE SHUNTS

A non-inductive resistance shunted across the primary windings of a current transformer has the same general effect as a secondary non-inductive shunt, viz., it increases the ratio and reduces the phase angle. The effect on ratio is proportional to the total equivalent resistance of the transformer and burden in terms of the primary circuit, and the effect on phase angle is proportional to the total equivalent reactance. If the power factor of the primary and secondary windings is less than that of the burden, as is frequently the case, the effect on phase angle as compared to the effect on ratio will be greater than with a secondary non-inductive shunt. This is generally to be desired.

A primary non-inductive shunt increases the ratio by approximately $\frac{R_t}{r_c} \times 100$ per cent., and reduces the phase angle by an angle $\tan^{-1} \frac{X_t}{r_c}$, where R_t and X_t are respectively the total equivalent resistance and reactance of the transformer and burden in terms of the primary, and r_c is the resistance of the primary non-inductive shunt. These formulae are approximate for reasons similar to those previously mentioned in connection with secondary non-inductive shunts.

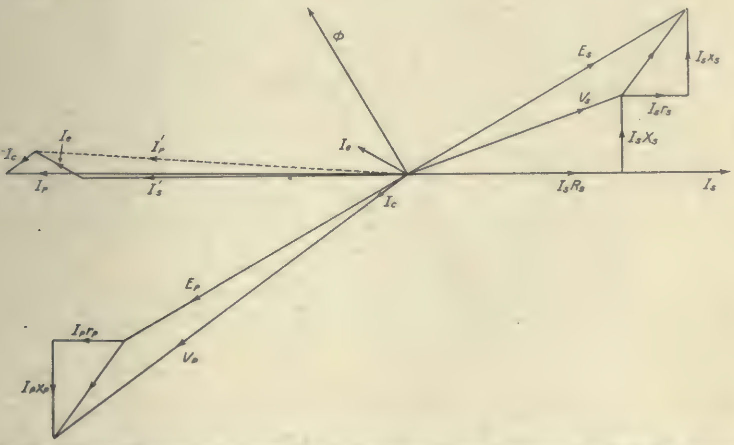

Fig. 6.—Vector diagram of a current transformer with non-inductive shunt in the primary.

Fig. 6 is a vector diagram illustrating the effect of a primary non-inductive shunt. By comparison with Fig. 2, the notation of which is similar, the difference between the effects of primary and secondary non-inductive shunts will be observed.

For illustration actual ratio and phase angle curves of a current transformer with and without a primary shunt are given in Fig. 7. Curves *B* and *C* are the same as curves *B* and *C* in Fig. 1, the transformer being the same and conditions the same. Curves *A* are obtained with a 4,000 ohm primary shunt with 223 turns in the secondary. As regards ratio, curve *A* is not as good as curve *B*, but the phase angle is reduced to the minimum. If 222 secondary turns are used with a 4,000 ohm primary shunt, curve *D* results, but *D* is not as good as *B*, particularly near rated current. Cases such as this will occur in which the reduction of phase angle error is accomplished at the expense of ratio no matter how the turns-compensation is adjusted. Some other kind of shunt would be necessary in such cases, for instance, a secondary non-inductive or primary or secondary condensive shunt or some combination of them. As an alternative some resistance may be deliberately added in series with the secondary burden and 222 secondary turns used with a primary non-inductive shunt of resistance which might need to be somewhat less than 4,000 ohms. The increased burden would tend to bring ratio curve *D* up to the desired position and the simultaneous tendency to increase the phase angle would be counteracted by a change in the value of resistance of the shunt.

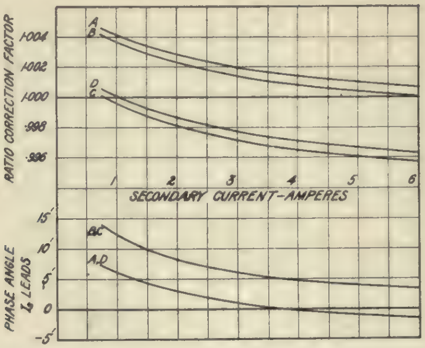

Fig. 7.—Curves showing effect of a non-inductive shunt in the primary. *A*, sec. turns 223, prim. non-inductive shunt of 4000 ohms. *B* and *C*, same as in Fig. 1 (no primary shunt). *D*, sec. turns 222, prim. non-inductive shunt of 4000 ohms.

The calculated change in ratio due to the shunt in Fig. 7 is $\frac{R_t}{r_c} \times 100 = \frac{2.48}{4000} \times 100 = 0.06$ per cent. The calculated change in phase angle is $\tan^{-1} \frac{6.87}{4000} = \tan^{-1} 0.0017 = 6$ minutes. Observed changes are given in Table IV.

A primary non-inductive shunt has another kind of advantage, namely, that it serves as a by-pass for high frequency current surges in

TABLE IV

Secondary amperes	1	2	3	4	5	6	Average observed change	Calculated change
Per cent. ratio change	0.04	0.04	0.04	0.05	0.06	0.06	0.05	0.06
Phase-angle change in minutes	6′	5′	5′	5′	5′	4½′	5′	6′

the main line which might otherwise injure the insulation of the current transformer.*

PRIMARY CONDENSIVE SHUNTS

A condenser connected across the primary terminals of a current transformer has qualitatively the same effect on the ratio and phase angle as one connected across the secondary. The ratio is decreased $X_t C\omega \times 100$ per cent., and the phase angle is reduced by an angle $\tan^{-1} R_t C\omega$, approximately. As an example Fig. 8 again reproduces curves A, B and C of Fig. 1, while curves D represent the ratio and phase angle with a 2 microfarad primary shunt, with 224 secondary turns which is also the number of primary turns. The calculated effect on ratio is

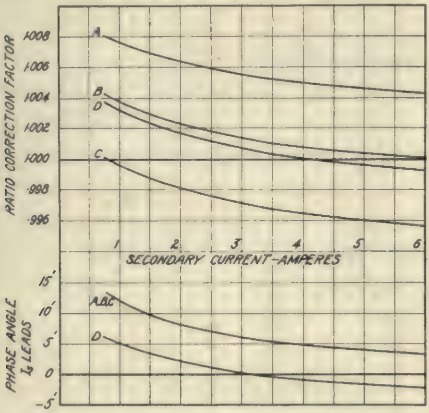

Fig. 8.—Curves showing effect of a condensive shunt in the primary. A, B and C, same as in Fig. 1. D, sec. turns 224, primary condensive shunt of 2 m.f.d.

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*Voltage Rises in Current Transformers, E. Wirz and Schweiz. Elektrotek-Verein, Bull. 7, p. 121, July, 1915.

Protection of Current Transformers against Voltage Rises, E. Wirz, Elekt-Zeits., 36, p. 450 and 467, Sept., 1915.

Protection of Current Transformers against Voltage Rises, E. Wirz and Geweche, Elekt.-Zeits., 37, p. 69, Feb., 1916.

Insuring Protection of Current Transformers, V. Karapetoff, Electrical World, 73, p. 473, 1910.

$6.87\times2\times10^{-6}\times377\times100$ or 0.51 per cent., and on phase angle $\tan^{-1}2.48\times2\times10^{-6}\times377$ or $\tan^{-1}0.00187$, *i.e.* $6\frac{1}{2}$ minutes. The observed changes, *i.e.* the differences between curves *A* and *D* are tabulated in Table V.

TABLE V

Secondary amperes	1	2	3	4	5	6	Average observed change	Calculated change
Per cent. ratio change	0.42	0.46	0.48	0.48	0.48	0.48	0.47	0.51
Phase-angle change in minutes	$7\frac{1}{2}'$	$6'$	$6'$	$6'$	$5\frac{1}{2}'$	$5\frac{1}{2}'$	$6'$	$6\frac{1}{2}'$

In curve *D* we have the peculiar condition that, although the transformer has equal primary and secondary turns, the true ratio falls below the turn ratio, or the secondary current is greater than the line current above 4 amperes. This is due, of course, to a partial resonance effect between the condenser connected across the primary and the reactance of the transformer and burden.

In current transformers of high ratio the primary impedance is low and the capacity required for a primary condensive shunt might be quite high. It would then be more economical to use a primary non-inductive shunt.

The primary condensive shunt as a corrective device is subject to disturbing effects of harmonics and frequency variation. It will pass high frequency current surges even more readily than the non-inductive shunt.

COMBINATIONS OF SHUNTS

Any or all of the shunts herein mentioned may be used simultaneously, and the effects of each will be added to those of the others, and may be calculated from the given formulae. The accuracy of the calculations depends upon the extent to which the assumptions on which the formulae are based, are realized.

SUMMARY

1. Primary and secondary non-inductive shunts tend to increase the ratio of a current transformer and reduce the angle of lead of the secondary current with regard to the primary current.

2. Primary and secondary condensive shunts tend to decrease the ratio of a current transformer and reduce the angle of lead of the secondary current with regard to the primary current.

3. With suitable turns-compensation one or more of the above mentioned shunts may be used to reduce the ratio and phase angle errors of a current transformer operating with fixed burden and frequency. The constants of the shunts may be calculated approximately from the formulae given if the required changes in the ratio and phase angle and the constants of the transformer and burden are known.

4. Condensive shunts may, under certain conditions, be useful in current transformers without fixed secondary burden.

5. Inductive shunts are not generally useful for reducing current transformer errors.

6. Primary shunts have the advantage of protecting the transformer and burden against high frequency current surges. Secondary shunts tend to protect only the secondary burden.

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